**Using the CAPM – Chapt. 8 in BKM**

The realized return for an asset is equal to its expected return plus returns generated from unanticipated macro events and unanticipated firm-specific events.

Ri = E(Ri) + mi + ei

mi = impact of unanticipated macro events

ei = impact of unanticipated firm-specific events

E(mi) = 0

E(ei) = 0

Different firms have different sensitivities to macro events.

F = macroeconomic factor (preferably the unexpected change in that factor)

βi = sensitivity of firm i to that factor (percentage change in return of firm i (on average) resulting from an unexpected one percent change in the factor)

mi = βiF (impact = sensitivity to effect times level of effect)

Example: F (the macroeconomic factor) is unexpected changes in the price of oil

The price of oil goes up unexpectedly (or more than was expected)

Exxon Mobil has a strong positive price change

United Airlines has a strong negative price change

Microsoft has very little price change

The sensitivity (beta) of XOM to this factor is strongly positive

The sensitivity (beta) of UAL to this factor is strongly negative  
The sensitivity (beta) of MSFT to this factor is either weakly positive, weakly negative, or zero.

If the price of oil does exactly what was expected, F=0 and the actual return for the stock is its expected return plus any firm-specific factors.

Single Factor Model – Only one macro factor

Ri = E(Ri) + βiF + ei -- a single factor model

A single factor model means that there is only one macroeconomic factor that influences the returns of all assets.

Or, we can think of it as all the various macro factors being wrapped up in this one factor

In a one-factor model, everything else that influences the return of an asset is firm-specific (not macroeconomic). All firm-specific factors are uncorrelated with each other and are uncorrelated with the macro factor. That means the firm-specific factors are diversifiable.

The CAPM is a single factor model where the excess return on the market is the single factor. Note that excess return means the return that is greater than or less than the risk-free rate of return.

The excess return on the market wraps up the effect of all the different macro factors into one factor

In the CAPM single-factor model, the excess return of an asset is the sum of three components:

Ri – Rf = αi + βi (Rm – Rf) + ei

αi = excess return if the market’s excess return is zero

βi = responsiveness of asset i to the (one) factor

(Rm – Rf) = excess return of the market. This is the one factor.

ei = impact of unanticipated firm-specific events (as before)

Why use excess returns? Because the risk-free rate of return varies over time, and whether you “should have” put your money in the market or in the risk-free asset, can only be determined by looking at how the market did in comparison to the risk-free rate at any point in time.

If risk-free rate is 1% 🡪 4% market return is good

If risk-free rate is 7% 🡪 4% market return is bad

A convenience of the single factor model is that it says that the covariance between two stocks is due to the influence of the single common factor on each of them. This means that we don’t have to worry about the covariances between individual stocks, only between each individual stock and the single factor (we don’t need to build a giant var/cov matrix!).

Given the single-factor model where (Rm – Rf) is the factor, we can write a univariate regression equation with Ri – Rf as the dependent (Y) variable and RM – Rf as the independent (X) variable. This will allow us to learn how the independent variable influences the dependent variable:

β will measure the percentage effect in the dependent variable that is caused by a 1 percent change in the independent variable (the slope of the regression line).

And α will be the excess return on the security when the market’s excess return is zero (the Y intercept)

Ri – Rf = α + β (Rm – Rf) + e

It is important to remember that such a regression has several underlying assumptions:

1. The relationship between the dependent variable and the independent variable (measured by β) is constant over time.
2. Unless we are using the entire population of returns, our sample will only yield point estimates of α and β. From there, we will need to construct confidence intervals for their true values using standard errors.
3. There is a linear relationship between the dependent and independent variables.
4. The residual values (e) are uncorrelated with each other

It is fairly easy to do a regression with excess returns on security i as the Y variable and excess returns on the market as the X variable. Our Excel output will give us estimates of the “true” alpha and beta values along with their standard errors.

The residuals (e) show the level of firm-specific risk. This might be quite high for an individual stock, but since E(e) = 0, when we have a reasonably large portfolio of stocks they should all cancel each other out (for each firm that has unexpected bad firm-specific news on a given day, another firm will have unexpected good firm-specific news).

In order to do the regression, we need a proxy for the market as well as a proxy for the risk-free rate.

Note that if the risk-free rate is constant over time, our estimates of α and β will be the same whether we use excess returns (subtracting Rf) in our calculations, or absolute returns. So if Rf is fairly constant over the time of our sample, we can disregard it in the regression. This is often done in practice, as a matter of convenience. However, we will learn how to use excess returns here.

If we use excess returns, the return on a U.S. Treasury instrument is usually used as a proxy for the risk-free rate. Academicians tend to use three-month treasury bill returns while practicioners will more often use returns on the 10-year note.

The most commonly used proxy for the market is the S&P 500. However, other proxies such as the Wilshire 500 or a Global Index can be used.

Be sure to realize though that the S&P 500 is **not** the market portfolio. It contains no bonds, no commodities, no real estate, and less than 40% of the market value of the world’s stocks. However, it is a market-value-weighted index, and its historic values are easily obtained.

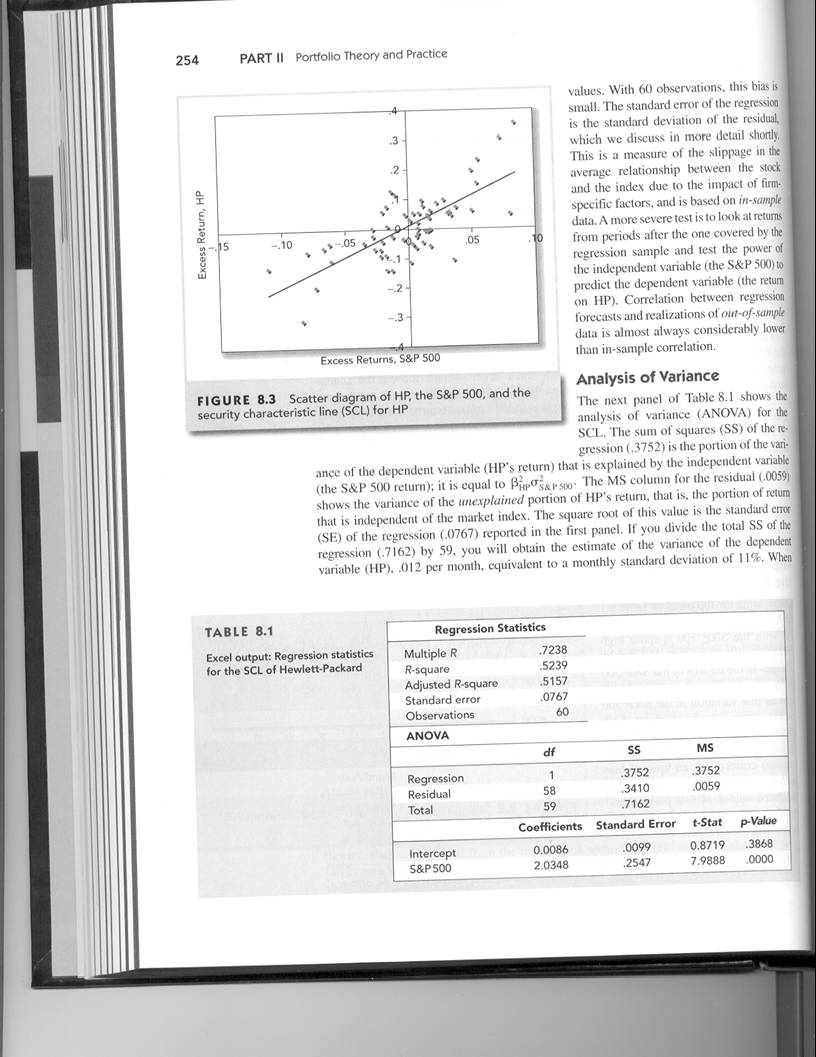
We also need to determine the frequency of our data (daily, monthly, annual, etc.) and how far back we should go to collect our data.

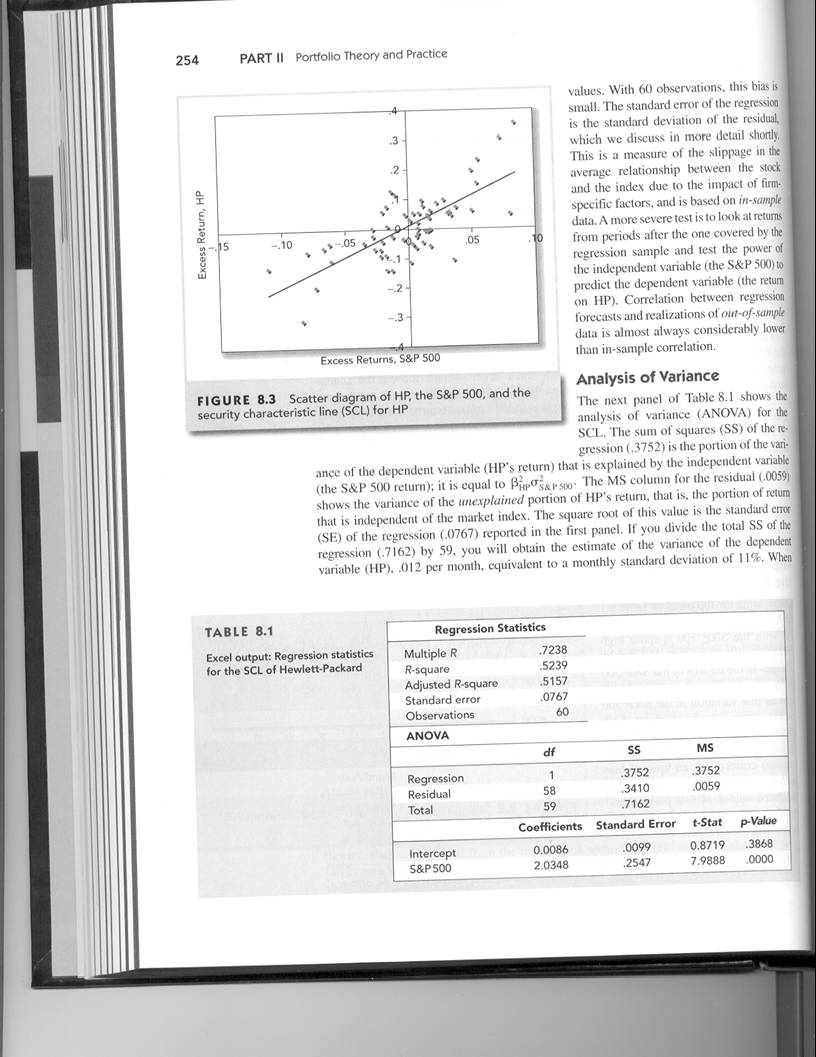
Going further back gives us more data points which improves the accuracy of our estimate (remember that we are using a sample of returns to estimate α and β). But, going back further also subjects us to an increased possibility that these values have in-fact changed over time. Remember that what we are ultimately interested in is an estimate of what these values will be going forward.

Using daily returns causes problems with the bid-ask bounce, perhaps giving the indication that returns were (in absolute value) larger than they actually were.

Various academic studies have concluded that the best balance is to use monthly data going back five years (60 data points). In practice, this is what is generally done, though some analysts use weekly data going back two years (about 104 data points).

Here is an example from a few years ago, of an estimate of the beta for HP stock.





For HP, the estimate of its beta is 2.0348 and the estimate of its alpha is 0.0086.

The t-statistic tells us how many standard errors the coefficient is from our null hypothesis. The default null hypothesis in Excel is that the coefficient is zero. So we obtain the t-stat by subtracting the null hypothesis from the coefficient and dividing that number by the standard error. A t-stat greater than 1.96 indicates that there is a 95% or more chance that the true value is different from the null hypothesis.

We easily reject the null hypothesis that HP’s beta is equal to zero and can safely conclude that its true beta is positive (greater than zero)

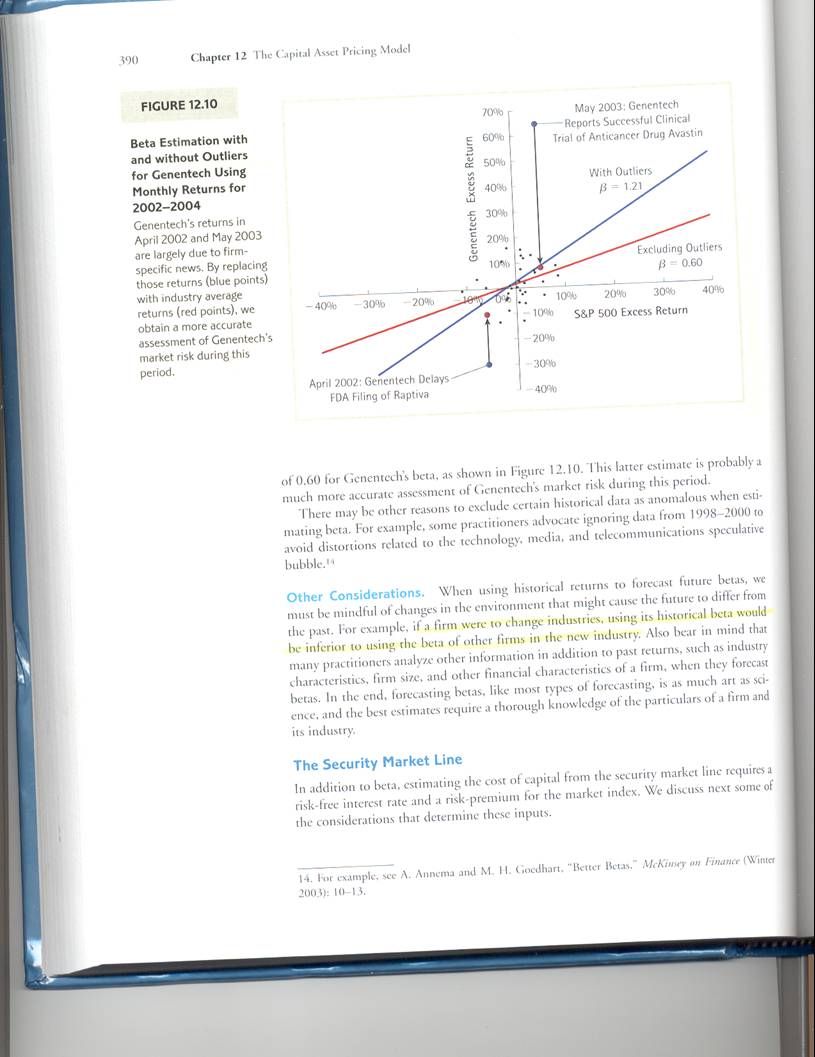
But we are not really interested in whether HP’s beta is greater than or equal to zero. We are more interested in if it is greater than one (since one is the average value for beta). So we subtract one from the coefficient and divide that by the standard error to get a t-stat of 4.07 – leading us to conclude that it is extremely probable that HP’s true beta is greater than one – that it has a higher than average beta.

Establishing a 95% confidence interval around the estimate of beta (adding and subtracting 1.96 standard errors from the point estimate) gives us a range of 1.54 – 2.53 for within which we are 95% sure the true beta lies (assuming it is stable over time). However, since the betas of stocks are notoriously not constant over time, this range can be better used as an initial unconditional forecast of what the stock’s beta will be next year. You must then use your knowledge of the firm, the industry and the market to make any adjustments to this value that you feel are appropriate.

Many practitioners will average the historical beta with an industry beta or with 1.0, since betas have been shown to migrate towards 1.0 and towards their industry average over time (mean reverting). These are sometimes called adjusted betas.

Merrill Lynch (and others) uses (1/3) (1) + (2/3) (βhistorical) = βAdj

You may also want to observe the effect of outliers and consider removing them from your sample:



Back to our single-factor model: Ri – Rf = αi + βi (Rm – Rf) + ei

If you take expectations of both sides of the equation and add Rf to each side as well, you end up with the traditional CAPM.

Note that the expected value of an error term is always zero and the expected value of alpha is zero because the sum of all risky assets IS the market, therefore the average Y axis intercept has to be zero.

It is also not uncommon to assume that any past positive alphas will be competed away in the future as smart investors like you find them and then bid up the price of the stock.

The CAPM: E(Ri) = E(Rf) + E(βi) (E(Rm – Rf))

Since interest rates are determined at time zero, E(Rf) = Rf, the current risk-free rate

And if we continue with the assumption that beta is constant over time, E(βi) = βi

For simplicity, we often write (E(Rm – Rf)) as (Rm – Rf)

Making the CAPM: E(Ri) = Rf + βi (Rm – Rf)

We usually treat the market risk premium (Rm – Rf) as one value rather than two. This is because it is really not a function of the market, but of investors’ attitudes towards risk.

The market risk premium answers the question: How much extra return must the market offer investors to entice the average investor to move his/her money from the safety of a risk-free investment to the (more risky) market?

There are several methods we can use to come up with an estimate of the market risk premium (sometimes called the equity premium)

1. Use past data

a. Assume that the average realized equity premium in the past equals the current required equity premium. This is the most often used approach. The problem with this is how far in the past you should go and how you should measure an average.

b. Look at the equity premium as a function of the market dividend yield (or some other factor), regress past equity premiums on past market dividend yields to get an alpha and beta estimate. Insert today’s dividend yield into the equation to forecast the expected future equity premium

2. Use a philosophical prediction: Ask yourself how much of a return is required to induce investors to switch from a risk-free asset to equities. Base your answer on where you see current investor sentiment.

3. Conduct a survey: Ask knowledgeable people (or average Joes) what they expect the equity premium will be going forward. Average the results or use the ones that make the most sense to you.

Finally, we must come up with a risk-free rate. This should be the **current** risk-free rate since it is forward looking.

Nothing is truly risk-free (standard deviation of zero), so we use a U.S. Treasury YTM as our proxy.

The maturity is often chosen based on the expected holding period of our investment. Academicians tend to use the three-month yield, but analysts are more likely to use the 10-year yield.

Can we test the CAPM to see if it works?

Not precisely – for several reasons

CAPM says that the market portfolio is MVE.

We can’t construct the true market portfolio

So, we must use a proxy like the S&P 500.

But, if the proxy is MVE, that doesn’t prove that the market portfolio is also MVE

And if the proxy isn’t MVE, that doesn’t prove that the market portfolio isn’t MVE

The dependent variable in the CAPM is *Expected* Return, how can you measure this? You can’t observe expectations, only actual returns.

Reasonable people can disagree about how beta should be measured, what a good estimate of the market risk premium should be, and even what the risk free rate is.

Thus, two people can observe the same data and come up with two different expected returns using the same pricing model (the CAPM).

Not only that – as we will see in the next section, the CAPM isn’t the **only** pricing model.