**Hedging and Immunizing with Duration**

1. You manage a pension plan with $200 million in assets that have a duration of 10 years and $150 million in liabilities with a duration of 4 years.
   1. Calculate your duration gap
   2. Are you hurt if interest rates go up, or if they go down?
   3. Suppose you want to execute a macro hedge using T-bond futures contracts traded on the CBOT. Do you want to go short or long?
   4. Suppose that the cheapest to deliver bond has a duration of 25.3 years. How many contracts will you short/long if you want to hedge your position as completely as possible?
2. You have been hired as a consultant by Steady-State Life Insurance Company. Steady-State specializes in the sale of Guaranteed Investment Contracts (GICs) which guarantee clients a specific dollar amount on a specific date in return for either a single up-front premium payment or a series of monthly payments.

Steady-State has just sold a GIC which guarantees a payout of $1.52 million on July 1, 2034. Steady-State needs to purchase a high quality bond that will assure them of having the $1.52 million on 7/1/34. They are willing to spend up to $950,000 on a bond to fund this GIC, and plan to buy it today, July 1, 2024.

Currently, the yield curve is flat so that the YTM of every bond with the credit-quality Steady-State is considering is currently 5.00%. You expect the yield curve to remain flat for at least the next 15 years, even though interest rates could go up or go down during that time.

Your job is to recommend a bond to purchase which will assure Steady-State of having the necessary $1.52 million in 10 years no matter how high or low interest rates might go between now and then. This is the only objective – make sure that the investment will grow to at least $1.52 million on July 1, 2034.

Steady-State has asked you to consider the following three corporate bonds that meet their standards for credit quality and liquidity. Each bond makes semi-annual coupon payments.

**Bond #1**

Face Value: $1 million

Coupon Rate: 4.25%

Maturity: July 1, 2034

**Bond #2**

Face Value: $1 million

Coupon Rate: 4.25%

Maturity: July 1, 2037

**Bond #3**

Face Value: $1 million

Coupon Rate: 4.375%

Maturity: July 1, 2039

Please answer the following:

1. Find the current price for each bond and ensure that it is less than $950,000
2. Calculate the duration of each bond
3. For each bond, determine how much money you will have on July 1, 2034 under each of the following three scenarios:
4. Interest rates stay at 5% (2.5% semiannually) for at least the next 15 years
5. Tomorrow, after you have purchased the bond, interest rates immediately go up to 6% and stay there for the next 15 years
6. Tomorrow, after you have purchased the bond, interest rates immediately go down to 4% and stay there for the next 15 years

Make note of the fact that Bond #1 matures when you need the $1.52 million, but Bond #2 and Bond #3 must be sold prior to maturity (sold on 7/1/34). Of course, they will sell for the present value of their remaining cash flows at that time.

Also note that all coupon payments will be reinvested at the prevailing interest rate when they are received.

So, for each bond, on 7/1/34, you will have the future value of the reinvested coupon payments plus either the face value of the bond (#1) or the sale price of the bond (#2 and #3).

1. Present a recommendation of which of the three bonds Steady-State should buy, and why.
2. Explain how an understanding of duration allows you to pick the correct bond without doing any of these calculations.
3. The correct bond will provide Steady-State with **more** than the needed $1.52 million if interest rates go up or if they go down. Explain what property of this (correct) bond causes this and calculate the value of that property for both the bond and the GIC to prove it.