**The Capital Asset Pricing Model**

**From Bodie, Kane and Marcus Chapter 9**

**Introduction**

Modern Portfolio Theory (which we learned about in the Chapter 7 notes) provides us with tools to select optimal portfolios *given* expected returns, variances and covariances. Where do we get these inputs?

The variances of assets and the covariances (or correlations) between assets are often assumed to remain fairly constant over time. Thus, we will usually calculate what these values have been in the past, make any adjustments that we think should be made to reflect changes that are occurring over time, and then use those numbers.

However, we must be certain to remember that what we are looking for is *future* variances (or standard deviations) and correlations. Since no one knows the future for certain, we are actually determining *expected* standard deviations just as much as we are determining *expected* returns. Just because we drop the “expected” term from standard deviations (implying that they are constant over time), doesn’t mean that we aren’t indeed forming expectations rather than calculating certainties.

With standard deviations and correlations, the best place to start forming our expectations is by calculating what they have been in the (fairly) recent past. This is only the starting point though – and should not always be considered to be the ending point. Careful evaluation of the individual securities and the economy may cause the analyst to conclude that future standard deviations and correlations may be different from what they were in the past. For this class though, we will calculate past standard deviations and correlations and use them as our expectations for the future.

Expected Returns however, are **absolutely not** constant over time and should never be found by calculating the actual returns of risky assets in the past and extrapolating those returns into the future. There is no reason, for example, to believe that because a stock has returned an average return of -3% to investors over the past five years, that it will produce a negative return next year. If that was the consensus view among investors, who would buy it?

We need *Asset Pricing Models* to help us determine expected returns.

Asset Pricing Models can be *theoretical* (derived rigorously from fundamental principles), or *empirical* (based on observed relationships in historical data on asset returns).

The most well-known asset pricing model is The Capital Asset Pricing Model (CAPM). The CAPM is a theoretical model, one of the simplest and earliest models, and by far, the most widely used model.

Not everyone feels that the CAPM is the best (or truest) asset pricing model, but a sound understanding of it is essential for everyone who studies finance as well as those who want to study more advanced asset pricing models.

**Assumptions underlying the CAPM**

1. Investors are in *perfect competition* with each other. This means that no single investor has the ability to influence prices with her/his trades.
2. All investors have the same one-period investment horizon. The CAPM does not say whether this period is one year, one month or one week, but there is only time zero and time one – there is no time two.
3. All investors are risk averse, and *the only things* they care about are the expected return of their portfolio its standard deviation.
4. All investors have homogeneous expectations. This means all investors calculate the same E(R) and σ for different securities, and the correlations between pairs of securities. Everyone looks at the same information and comes up with the same conclusion.
5. Markets are frictionless. This is a collection of assumptions that includes:
	1. Investors can borrow or lend any amount of money at the risk-free rate
	2. All investments are infinitely divisible, which means you can buy or sell any fraction of any asset
	3. There are no taxes or transactions costs
6. There is no inflation or changes in interest rates.
7. Capital Markets are in equilibrium. This means that all securities are properly priced taking into account their risk and that prices adjust instantly to any new information.

These assumptions are clearly very stringent and nobody is naïve enough to believe that these assumptions are actually true. In fact, if they were true, we wouldn’t have to call them assumptions. However, the nature of these assumptions is the price we pay for the simple, yet powerful, results of this theory. Many of these assumptions can be, and have been, relaxed. In most cases the basic intuition is still the same (even though the algebra and calculations get a lot messier).

**The Market Portfolio**

The above assumptions mean that *every* investor solves an identical constrained optimization problem, obtains the same efficient frontier, and decides to invest in a combination of:

* The risk-free asset
* The MVE (optimal risky) portfolio

The key question to ask is: If every investor holds the *same* risky portfolio (the MVE), what assets make up the MVE?

The answer to this question is the key to the CAPM.

It can be proven that when the opportunity set contains **all** risky assets and all combinations of risky assets, the MVE **must** be the market portfolio.Intuitively, this makes sense, because if every investor chooses to invest in the same MVE, the MVE must contain all risky assets, or otherwise there would be risky assets that no one would own and their price would fall to zero (when, of course, investors would begin to add it to their portfolios).

The market portfolio is a portfolio of all risky assets in the economy, with each asset in proportion to its market value. i.e. the weight of each asset i in the market portfolio is:



This means that under these assumptions, the only portfolio held by any and all investors (in addition to the risk-free asset) is the market portfolio.

So, under the CAPM assumptions, we don’t need to solve for the MVE as we did when we were looking at only 8 risky assets (this is good news since Solver probably won’t accommodate every risky asset in the economy).

We *know* that the optimal portfolio (the MVE) will bethe market portfolio.

Let’s say some new information comes out that makes you believe that a particular stock is underpriced, so you determine that you should hold a bit more of it in your MVE portfolio.

But (under the CAPM assumptions), at the exact same moment, every other investor is coming up with the exact same conclusion, and everyone will add a bit more of it to *their* MVE portfolios.

The aggregate effect of this will be to increase the proportion of this stock in the *market portfolio*.

Since the market is in equilibriumat all times (another one of the assumptions), all these changes take place instantaneously. There is no opportunity for any single investor to “beat the market”. This suggests that our risk-averse investors will do the following:

Step 1: Their investment in risky assets is only into the MVE portfolio. This is their *investment decision*. Everyone will reach the same investment decision.

Step 2: Now, to attain their preferred point along the CAL, they will decide how much to borrowor lendat the risk-free rate. This is their *financing decision*. Everyone will reach different financing decisions, based on their coefficients of risk aversion (A) and what the risk-free rate is.

**The Capital Market Line**

The Capital Market Line (CML) is nothing but our old Capital Allocation Line (CAL) with the knowledge that the MVE portfolio is the market portfolio.



rf

Market

Portfolio

Since the CML is further northwest than any other asset, all efficient portfolios lie along the CML, and are combinations of Rf and the Market Portfolio.

Since the only risky asset that investors will invest in is the market portfolio, the definition of risk for an individual asset will not be its standard deviation, but how it affects the standard deviation of the market portfolio.

The market portfolio is our “team”, and we evaluate each member of our team (each individual asset), not by how they perform on their own, but by how they contribute to the performance of the team. So how does an individual asset affect the standard deviation of the market portfolio?

**The Pricing Model**

CAPM: The expected return of any asset i satisfies the following:

E(Ri) = Rf + βi (RM – Rf) where βi =  =  =  

**What the CAPM is telling us**

1. Covariance (with the market portfolio), not standard deviation, is the appropriate measure of risk for an individual asset.

Why? Because the covariance of an asset with a portfolio can be interpreted as the marginal variance of the portfolio.

To see this, look at a 100 x 100 variance/covariance matrix and see what happens when you add risky asset # 101. The marginal contribution of the asset to the portfolio’s variance is 200 times more dependent on the covariance of asset #101 with the other assets than with the variance of asset #101.

Beta (covariance with the market standardized by the variance of the market) is thus the relevant measure of risk.

1. High beta assets have high expected returns

In fact, according to the CAPM, there is a linear relationship between beta and expected return.

What is the economic intuition behind this result? High-beta assets have high returns when the overall market return is high i.e. they pay off well when you least need the money. Conversely, when you need the money most (which is when the overall market is doing badly), these assets fare poorly. So, in equilibrium, these assets have to offer high expected returns in order for investors to hold them.

Additionally, high-beta assets have a high covariance with the market. If investors are holding the market portfolio, this means that when they add a high-beta asset to their portfolio, they are increasing the standard deviation of their portfolio – thus making it more risky.

Investors require a high expected return as compensation for holding stocks that increase the risk (standard deviation) of their portfolio.

1. Beta alone determines expected returns

Once beta is taken into account, no other factors determine expected return.

Beta is the sole factor affecting security prices and returns. That makes the CAPM a single-factor model. We will contrast this with multi-factor models later.

1. Diversifiable risk is not priced

The best way to see this is to consider an asset that has a high standard deviation, but a zero correlation with the market portfolio.

The CAPM says that this asset, in spite of its high volatility, will have an expected return equal to the risk-free rate, as its beta is zero.

This means all the risk of this stock is idiosyncratic (unique), which means it can be diversified away.

Since diversifying is costless (one can easily buy into a no-load mutual fund or an ETF), investors are not compensated for such company-specific risk.

The only risk that is priced is systematic risk (market risk), which is measured by beta.

**Additional Notes on beta**

The definition of beta is βi = 

It can also be written as βi = 

Beta is really a measure of the covariance of an asset’s returns with the returns of the market. It is simply standardized by the variance of the market to give us a more easily understood number (an average beta is 1.0, but an average covariance with the market can change based on the variance of the market and it is unlikely to be a round number).

Beta can also be understood as the sensitivity of the asset’s returns to the returns of the market – that is, the slope of the regression line when we regress the return of the asset on the return of the market (This is a simplified way to measure beta. We will see later on, that doing it this way assumes that the risk-free rate was unchanged over the life of the time you recorded your returns).

One should always remember that:

β of risk-free asset = 0, as the risk-free asset has a covariance (and correlation) of zero with any other asset, including the market portfolio.

β of the market portfolio= Cov(rm,rm)/Var(rm) = Var(rm)/Var(rm) = 1, since the covariance of any asset with itself is the variance of that asset.