**Risk and Risk-Aversion Part 1**

**From Bodie, Kane and Marcus Chapter 6**

Overview: Very early in your first finance class, you learned that the value (price) of any financial asset is the present value of its future cash flows.

P0 = C0 + C1 + C2 + C3 + …

 1+r (1+r)2 (1+r)3

A class such as Financial Statement Analysis is all about how to forecast cash flows (the numerator in this formula). This course is all about how to determine the correct discount rate to use (the denominator in this formula).

Modern Portfolio Theory – the generally accepted way that discount rates have been determined and portfolio managers have evaluated investments since the 1970s.

Start with a definition of risk. There is not a universally accepted definition, but I like the following one:

Risk means more things can happen than will happen

How do we quantifiably measure risk? There are several ways, but the traditional measure is Standard Deviation (in part, because it is easy to calculate).

Note though that not everyone agrees that this is the best way to measure risk.

**Risk and Return: The trade-off**

Consider the following choice facing an investor with $100,000 to invest. She has the choice of investing the $100,000 in a *risk-free* investment or a *risky* investment.

Possible payoffs of both investments are:

Risk-free investment

 100,000 105,000

Risky investment

 .6 150,000

 100,000

 .4 80,000

The (simple) return on the risk-free investment is:

Rf = 105,000 – 100,000 = 105,000 – 1 = 5%

 100,000 100,000

The *expected* return on the risky investment is:

E(R) = .6[(150,000/100,000) – 1] + .4[(80,000/100,000) – 1] = 22%

Which investment should our investor choose? Which would you choose?

To try and make a decision, Modern Portfolio Theory says that we proceed in three steps:

Step 1: Calculate the *risk premium* or *expected excess return* on the risky investment.

The risk premium on any risky asset is its expected return over and above the risk-free rate.

We can calculate this two ways: E(R – Rf) or E(R) – Rf

E(R – Rf) = .6(50% – 5%) + .4(-20% – 5% ) = 17%

E(R) – Rf = 22% – 5% = 17%

Step 2: Calculate the Standard Deviationof the risky investment. Of course, this is the square root of its variance, which is also measures risk.

The standard deviation of the risk-free investment is zero by definition

The variance of the risky investment is:

σ2 = .6(.5 - .22)2 + .4(-.2 - .22)2 = 0.1176

and its standard deviation is  = 34.29%

Step 3: We need to decide whether the 17% risk premium is commensurate with the 34.29% risk. That is, we need a way of telling us whether 17% is adequate compensation for this risk, more than adequate, or less than adequate. Of course, not everyone will come to the same conclusion as to whether it is adequate compensation or not, but we want a quantifiable, objective decision-model that will tell us what an individual will decide and why.

The whole field of *asset pricing* has developed (and continues to expand) to answer this one question. *Asset Pricing Models* like the CAPM are attempts to answer this question.

We start with the assumption that investors are expected utility of wealth maximizers. This means that they will choose the investments that give them the highest expected utility (happiness). Remember that an expected value is a statistical term.

Next, we need to make some assumptions regarding investors’ attitudes towards risk. How do they feel about risk? Do the like it, dislike it, or are they indifferent towards it? Can we assume that all investors feel the same way?

**The two risk-return assumptions of Modern Portfolio Theory**

Assumption 1: Investors prefer more wealth to less wealth.

Assumption 2: Utility of wealth increases with wealth at a decreasing rate. In other words, it has a declining marginal utility.

Assumption 1 is simple enough. We all like more wealth. Economists call it non-satiation.

What about Assumption 2? Who is more excited about being given a $10,000 raise – you or Bill Gates?

These two assumptions lead us to the conclusion that investors are risk-averse. Their expected utility of wealth from a fair gamble is less than their certain utility of wealth if they don’t accept the gamble. Thus, they will not make the gamble unless they are offered a risk premium as compensation.

To better understand, consider the following gamble:

$150,000

H (0.5)

Flip a coin

$50,000

T (0.5)

Expected value of gamble, E(gamble) = (0.5×150,000) + (0.5×50,000) =$100,000

How much are you willing to pay as an entry fee to take on this gamble – how much makes you indifferent between doing it and not doing it?

If you are willing to pay more than $100,000, you are *risk-loving*

If you are willing to pay $100,000, but no more, you are *risk-neutral*

If you are not willing to pay $100,000, you are *risk-averse*

Risk aversion does not mean that investors are unwilling to accept risk. It means that investors wish to be *compensated for risk*. In the example above, there is no compensation (in the form of expected return) for paying $100,000 or more to play the game.

Note: In the above example, if you are willing to pay $80,000 to take on the gamble, but not a penny more (technically, if you are indifferent at $80,000), we say that $80,000 is the ***certainty equivalent***of the gamble for you. It is the *risk-free amount* where an investor is indifferent between accepting that risk-free amount and accepting the *risky* gamble.

Another example of risk-aversion:

Consider this common situation:

Fire

(1/1000)

Loss: 100,000; value=0

House

No fire

(999/1000)

Loss: 0; value=100,000

E(loss from fire) = (1/1000)×100,000 + (999/1000) ×0 = $100

We purchase insurance paying an amount I and eliminate the financial risk

100,000-I

Fire

(1/1000)

100,000-I

Value of house and insurance policy - both

No fire

(999/1000)

If you are willing to pay more than $100 for your fire insurance, you are risk averse. Most people will purchase the fire insurance.

But what about people who routinely pay $1 to play the lottery? The expected value of playing is generally less than $1. Does this mean that they are both risk-loving and risk-averse?

Modern Portfolio Theory explains this paradox saying that, when investors play with small amounts of money, they might act as risk-loving individuals, but they turn risk-averse when the stakes are materially high. Additionally, when there is a “game” involved, like the lottery, people are willing to pay a price for the enjoyment of playing the game. With the lottery, they are paying $1 for the opportunity to dream about becoming fabulously wealthy.

**Indifference curves**

The risk-return assumptions that we noted above are surprisingly powerful.

We can use these assumptions to draw indifference curves regarding investors’ preferences on risk and return.

An indifference curve is a plot of all risk-return combinations that give the investor the samelevel of utility. Each indifference curve represents a particular level of utility.

A set of typical indifference curves over risk and return looks like the following.

U1

U2

U3

Expected

Return, E(R)

Increasing utility

Standard Deviation, σ

Two points to be noted from this graph.

1. Investors prefer more return and less risk. i.e. investors want to be as much to the northwest of the above graph as they can be. That is why indifference curve U3 represents more utility than U2, which represents more utility than U1.
2. At lower levels of risk, it takes a small amount of return to tempt the investor to take a greater amount of risk. At higher levels of risk, this is reversed. This is the essence of risk aversion: investors want to be compensated for taking greater amounts of risk. That’s why the slope of the curve is steeper as you climb it.

 Now, let’s compare the following two investors:

More risk-averse investors have steeper indifference curves

Less risk-averse investors have flatter indifference curves

E(R)

19%

E(R)

14%

12%

 15% 20%

 15% 20%

σ

σ

**Utility Functions**

In economics, the standard way of representing people’s preferences over goods or investments is through a utility function.

The utility function returns a score of utility or “happiness” from a given investment that has a given expected return and standard deviation.

In this course, we will use the following utility function:

 U = E(r) – ½ A σ2

This is a quadraticutility function. Note that this is only one of many possible utility functions that we could have used. We will use this particular function because it possesses many (mathematically) nice properties and because it is used in your text. The appendix for chapter 6 in BKM gives some examples of other utility functions such as the lognormal utility function, which actually does a better job of modeling investors’ preferences, but is more difficult to work with.

The quantity A is called the coefficient of risk aversion. The higher A, the higher the investor’s risk aversion.

A>0: the investor is risk-averse

A=0: the investor is risk-neutral

A<0: the investor is risk-loving

How do we know what a person’s coefficient of risk aversion is?

We would have to present them with a number of investment choices and see which ones they prefer. If the investor is internally consistent with his/her choices (unfortunately, this is not always the case), we can determine how risk-averse the person is. The text gives some sample questions in chapter 6.

Let’s draw indifference curves from this utility function, and confirm that it represents a risk-averse investor.



These are curves for an investor with A=0.75. Three different indifference curves for the same investor. Note that these indifference curves demonstrate risk aversion as we understand it. Each indifference curve represents a particular level of utility, and increasing utility takes us towards the northwest frontier of the graph.

Each point (each combination of Expected Return and Standard Deviation) on the same curve gives this investor the same utility. He is equally happy with any two investments that are on the same indifference curve.

Let’s see how indifference curves look for three different investors with different levels of risk aversion.



I have picked here for the same utility level (U= 0.05), three investors with differing levels of risk aversion. As we saw before, more risk-averse investors have steeper indifference curves.

Getting back to our investment choices on page 1: what should our investor choose, the 5% risk-free investment, or the risky investment with an expected return of 22% and a standard deviation of 34.29%?

Let’s assume a coefficient of risk aversion of A=3.00. Then we plug the expected returns and standard deviations of the investments into our utility function and make a decision.

With A=3.00, the utility she gets from the risky investment is

U*risky* = .22 – [(1/2) (3) (.3429)2] = .0436 = 4.36%.

For the risk-free investment, the expected (risk-free) return is 5%, and the variance is zero. i.e. U*risk-free* = .05 – [(1/2) (3) (0)2] = 5%.

If our investor has a coefficient of risk aversion of 3.0, she would choose the risk-free investment because it gives her more utility.

If, however, our investor were less risk-averse and had a lower coefficient of risk aversion, say 2.0, she would choose the risky investment.

U*risky* = .22 – [(1/2) (2) (.3429)2] = 10.24%

vs. 5% for the risk-free return.

The moral of the story is: Investor choice depends on investor preferences which depends on just how risk averse the investor is.

Two of the nice features of this quadratic utility function are that the certainty equivalent equals the level of utility, and for a risk-free investment, the level of utility equals the rate of return.

Remember that the certainty equivalent is the risk-free rate of return that would make an investor indifferent between that certain return and a risky one. To be indifferent between these two investments, they must offer our investor the same amount of utility. However, this doesn’t necessarily mean that this particular risk-free rate of return is actually available. Note that in our example above, the investor with A=2 received utility of 10.24% from the risky investment. This means that her certainty equivalent was 10.24% (she would be indifferent between the risky investment and a certain 10.24% because each gives her the same utility). However, the risk-free rate is only 5% - there is no risk-free rate of 10.24% available to her (which is why, with a coefficient of risk aversion of 2.0, she prefers the risky investment)