**Finding the Efficient Frontier**

**Instructions for Students**

**General Instructions**

The objective of this assignment is to graph the efficient frontier for eight Exchange Traded Funds (ETFs) coming from five different asset classes under a variety of constraints and to use that information to decide what weightings you would recommend placing on each ETF and asset class in the portfolio.

At the end of these instructions, you will find a list of five asset classes along with the symbols for a number of different ETFs which track indexes within each of those asset classes. You may select any eight of these ETFs for this exercise.

The spreadsheet that accompanies these instructions will graph the efficient frontier for you after you have selected eight ETFs, a risk-free rate, and a market risk premium. The spreadsheet contains a macro which allows it to go to Yahoo! Finance (if your computer is connected to the internet) and find the adjusted closing prices of the ETFs you select, along with the S&P 500, for the most recent 61 months. From that data, the spreadsheet will automatically calculate the following for each of the eight ETFs: monthly returns, the annualized standard deviation of those returns, correlations and covariances with the other seven ETFs, a beta coefficient, and the expected return for that ETF using the Capital Asset Pricing Model (CAPM). In the CAPM calculations, you have the ability to select whatever risk-free rate (Rf) and market risk premium (RM-Rf) you choose.

The spreadsheet will then build a weighted variance/covariance matrix where it uses solver to find the smallest possible standard deviation for the eight-ETF portfolio with portfolio expected returns ranging from 2% to 13% (in increments of 1%). Each of these 12 solutions is listed on the last worksheet where they are automatically graphed in mean/variance space – building the familiar-looking hyperbola. Additionally, the spreadsheet calculates both the minimum variance portfolio (MVP) and the mean/variance efficient portfolio (MVE). The graph shows the location of each of the eight selected ETFs, the risk-free asset, the MVP, and the MVE.

The spreadsheet also gives you the opportunity to place various constraints on your portfolio. You can choose a “no-short-sales” constraint which restricts the weights on each ETF to be non-negative. You can also choose a “diversified” constraint which prevents any ETF from having a weight greater than what you choose. You can also choose to restrict the total weighting on four of the five asset classes (Foreign Equity, Fixed Income, Real Estate, and Commodities). Finally, for 17 of the traditionally more risky ETFs, you have the option to constrain any of their weights so that they are something less than or equal to 20%. The spreadsheet allows you to select any combination of the above constraints.

**Specific Instructions**

For this assignment, use a risk-free rate of 3% and a market risk premium of 5.7%. Select any eight ETFs from the list at the end of these instructions that you would like to include in your portfolio, and then do the following:

1. Enter the ticker symbols for the ETFs you have chosen in the gray-shaded cells
2. Do not place any constraints on the weights
3. Click the “Download” button
4. Study the expected returns and standard deviations of each of the ETFs as well as the correlation matrix
5. Print out the last page of the spreadsheet which includes the graph
6. Place a “no shorting” constraint by excluding negative weights for all ETFs.
7. Repeat steps 3-5 with the “no shorting” constraint
8. In addition to the “no shorting” constraint, select the “diversified” constraint which requires the weight of each ETF to be less than some amount (choose whatever amount you want).
9. Again, repeat steps 3-5
10. In addition to the “no shorting” and “diversified” constraints, if you consider any of the 17 ETFs in the blue cells to be particularly “risky” (use whatever definition of “risky” you feel is appropriate), limit their weights to be something less than 0.2 by selecting that ETF and a weight (less than 0.2) that you want to constrain that ETF to be less than or equal to. Please ignore India in row 44, however. It has the wrong ticker symbol.
11. Again, repeat steps 3-5.
12. Compare the four graphs that you printed in steps 5, 7, 9, and 11. Which graph(s) appears to offer investors the least portfolio risk (as measured by standard deviation) for the given expected returns? Which graph(s) seems to offer the greatest choice among portfolio expected returns? Why? Do the constraints you imposed decrease the standard deviation or increase it? What role do the correlations between the ETFs play in Excel’s selection of the optimal weights?
13. Make some changes in your choice of the eight ETFs and repeat steps 1-12. Did any of your conclusions in step 12 change with this new set of ETFs?
14. Choose the eight ETFs and the weights on those ETFs that you feel will give you the “best” portfolio if you are managing a hedge fund for a variety of individual and institutional investors. Feel free to define “best” however you want. Feel free to choose different asset classes and ETFs from those you used above. There is no right or wrong set of choices for this step, but please explain **why** you decided to select this particular combination of ETFs and their weights for your hedge fund.

**Efficient Frontier – List of ETFs**

**Asset Class ETF**

Domestic Equity

Large-Cap Value VTV

Large-Cap Growth IWF

Mid-Cap Value DVY

Mid-Cap Growth IJK

Small-Cap Value VBR

Small-Cap Growth IWO

Financial XLF

Health XLV

Energy IYE

Technology VGT

Utilities IDU

Foreign Equity

China FXI

India INPTF

Latin America EWZ

Europe EWG

Emerging Markets VWO

Fixed Income

Long-Term Bonds LQD

Intermediate Bonds BIV

Short-Term Bonds IGSB

TIPS TIP

Junk Bonds HYG

Real Estate

Domestic Real Estate VNQ

Global Real Estate IFGL

Commodities

Precious Metals IAU

Natural Resources VAW

Agricultural Commodities RJA