**Multifactor Models – from BKM Chapt. 10 and 13**

Single-factor Model: Ri = E(Ri) + βi F + ei

βi = sensitivity of asset i to the factor

F = the factor

CAPM is a single-factor model where the market’s excess return (above the risk-free rate) is a proxy for all systematic risks, wrapped up into one factor.

Separating risk into systematic and non-systematic components is a great idea. However, the market captures all systematic risk only if the market portfolio (or technically, our proxy for it) is the MVE – which means that everyone invests in it. However, it cannot be proved (nor disproved) that the market is the MVE; only that, given the assumptions of the CAPM, it should be. If the assumptions are relaxed, we are left with an untestable theory.

The market’s excess returns come from a number of macro-economic risk sources, each of which affect different firms differently. As we showed before, when oil prices unexpectedly increase (a risk source), it affects Exxon Mobil, United Airlines and Microsoft all differently.

A single factor model, with the excess market return being the single factor, implicitly assumes that every stock has the same relative sensitivity to each risk factor other than the market.

Multifactor models allow for the possibility that different stocks have different sensitivities to each of the separate factors in the model.

With a multifactor model, there will be **factor betas** for each risk factor. Each factor beta represents the sensitivity of that firm to that specific risk factor.

For example, if unexpected changes in oil prices is a risk factor, the factor beta (for that risk factor) for UAL might be -1.7, meaning that for every 1% increase in oil prices, UAL stock drops (on average) 1.7%.

With a multifactor model, there will also be **factor risk premiums** for each factor. In the same way that the risk premium for (Rm – Rf) in the CAPM was 5.7%, each factor’s risk premium will be the compensation that we receive in our expected return for bearing that particular risk (if the factor beta is 1.0).

Do not assume that all factors work the same way as (Rm – Rf). For example, with other factors, the average beta will probably not be 1.0. There is no reason to believe, for example, that if oil prices go up by 1%, this will cause the average stock to go up by exactly 1%. Also, just because (Rm – Rf) naturally gives us a positive risk premium, this doesn’t necessarily mean that all risk premiums are positive. Some can be negative!

To see this, let’s assume that we have a **2-factor model** where the two factors are 1) changes in GDP and 2) changes in interest rates.

RPX = the risk premium for factor X. e.g. RPRm-Rf = 5.7%

Leaving out the subscript i so that we can specify the betas and risk premiums more easily, you can see that this is just like the CAPM, but with two factors.

E(R) = Rf + βGDP(RPGDP) + βIR(RPIR)

Most firms have a positive sensitivity (beta) to changes in GDP. When GDP goes up, the return for most firms goes up and when GDP goes down, the returns for most firms goes down.

However, most firms have a negative sensitivity (beta) to changes in Interest Rates. When interest rates go up, most firms see their returns go down and when Interest Rates go down, most firms see their returns go up.

Stocks which have similar sensitivities (betas) to what most stocks do, will increase the risk of a portfolio (they don’t add much diversification). But stocks which have different sensitivities (betas) to what most stocks do, will decrease the risk of the portfolio (they add a lot of diversification).

Investors require a higher expected return for stocks that do not add much diversification to their portfolio. Those same investors are willing to accept a lower expected return for stocks that add a lot of diversification.

This means that a stock with a positive βGDP should have a higher expected return and a stock with a negative βGDP should have a lower expected return.

It also means that a stock with a negative βIR should have a higher expected return and a stock with a positive βIR should have a lower expected return.

For a stock with a positive beta to get a higher expected return, the risk premium must be positive. This is the case with the risk premium for GDP.

But for a stock with a negative beta to get a higher expected return, the risk premium must be negative. This is the case with the risk premium for Interest Rates.

So if the typical sensitivity (beta) to a risk factor is negative (as with changes in Interest Rates), the risk premium will be negative. If the typical sensitivity (beta) to a risk factor is positive (as with changes in GDP and with excess returns on the market), the risk premium will be positive.

Though we can often intuitively see whether a risk premium should be positive or negative, calculating factor risk premiums directly is not a simple task. It requires a two-stage regression that would be difficult to understand if you have not had a course in advanced econometrics. Making things even more challenging is the fact that risk premiums should be forward looking, so any estimation method that uses past results should not necessarily be taken as authoritative. Remember that for the CAPM, the risk premium for the only factor (Rm – Rf) is certainly not universally agreed upon.

Later in this lesson, we will learn how we can use factor portfolios to indirectly estimate factor risk premiums, and self-financing factor portfolios can be constructed to simplify matters by ensuring that all factor risk premiums are positive.

The intuition behind a factor risk premium is this: In a multifactor model, the risk premium for each factor is the risk premium for the portfolio if it has a factor beta of one for that factor and a factor beta of zero for all other factors. In other words, if this were the only factor, what would the stock’s risk premium be?

Let’s go back to our simple 2-factor model: E(R) = Rf + βGDP(RPGDP) + βIR(RPIR)

Suppose RPGDP = 6% and RPIR = -7%

Also, suppose we have a stock with positive sensitivity to changes in GDP (which is typical), but also has a positive sensitivity to changes in Interest Rates (which is unusual).

βGDP = 1.2 and βIR = 0.3

Let’s also suppose that the risk-free rate is 3%. Our 2-factor model will look like this:

E(R) = Rf + βGDP(RPGDP) + βIR(RPIR)

= 3% + (1.2)(6%) + (0.3)(-7%) = 8.1%

Investors require an increased expected return (above the risk-free rate) as compensation for bearing GDP risk, but are willing to accept a reduced expected return because the stock has an unusual positive sensitivity to Interest Rate risk. The impact of each factor on this stock is added to the risk-free rate to give us the expected return that investors will require

**Estimating Factor Betas**

Factor betas are estimated the same way that we earlier estimated the CAPM beta.

For an individual stock i, run a regression with Ri – Rf as the dependent variable and values for the factors over the same time period as your independent variables (note that this is now a multivariate regression rather than a univariate regression). Again, this is typically done over the past 60 months. You will get factor beta estimates with standard errors just as we did for the CAPM beta.

**Factor Portfolios**

If a portfolio of stocks is highly correlated with the risk factor, you can use its excess returns to estimate factor betas rather than the risk factor values themselves. Practically, this can make matters simpler when you want to determine is the *unexpected* *change* in the factor value.

For example, if your risk factor is unexpected changes in oil prices, your factor portfolio can be made up of a number of different securities whose returns you know to be highly positively correlated with unexpected changes in oil prices.

Now, instead of regressing the excess return of stock i on the change in the price of oil (which may or may not have been expected), you regress the excess return of stock i on the excess return of the factor portfolio (which is highly correlated with those changes).

An extension is to create a “self-financing” factor portfolio where you long securities that are highly positively correlated with the risk factor and short securities that are highly negatively correlated with the risk factor. This self-financing portfolio is now the difference between the returns on the two “sub” portfolios, so there is no need to subtract the risk-free rate.

The ability to form a factor portfolio from stocks which are correlated with the risk actually allows us to control for the risk without having to specifically identify what it is (as long as we know which stocks are affected by it).

It also allows us to use the long-run average return on the self-financing factor portfolio as our factor risk premium. This is similar to how we use the historic market risk premium of 5.7% as our estimate of the current market risk premium we have been using for the CAPM. This is the indirect method of coming up with factor risk premiums that we mentioned earlier.

**The APT and Multifactor Models**

The Arbitrage Pricing Theory (APT) was first developed by Robert Merton as the intertemporal CAPM, then put forth in its current form by Stephen Ross. The theoretical underpinnings of the APT are somewhat different from the CAPM, but it basically comes up with the same results – except that investors only need to hold “well-diversified” portfolios and not specifically the market portfolio, and that there are multiple risk factors, not simply one (as in the CAPM).

There is no monopoly on the number of risk factors, nor what they should be. Hundreds of investment firms have each put together their own proprietary multifactor models with risk factors and risk factor premiums that they feel will help them identify mispriced stocks.

**The Fama/French Three-Factor Model**

The Fama/French Three-Factor Model is probably the most famous multifactor model and came about as a result of what Fama and French showed in their famous 1992 paper.

Factor 1 = Excess returns on the Market Portfolio (usually using the S&P 500 as a proxy for the market portfolio)

Factor 2 = SMB = Return of a portfolio of small cap stocks minus the return of a portfolio of large cap stocks

FF separate firms into two portfolios based on the market value of their equity (market cap). One group (S) is below the median size of NYSE firms and the other group (B) is above that value. The factor portfolio shorts group B and uses the proceeds to purchase group S.

Factor 3 = HML = Return of a portfolio of high B/M ratio stocks minus the return of a portfolio of low B/M ratio stocks

All firms with book-to-market ratios less than the 30th percentile of NYSE firms form a value-weighted portfolio which are called L. Another portfolio called H is formed from all firms with book-to-market ratios greater than the 70th percentile of NYSE firms. The factor portfolio shorts group L and uses the proceeds to purchase group H.

Fama and French looked at ex-post data to find these factors.

They found what best fit the data.

They had no theoretical justification for picking them except that they worked.

You can find the returns on these factor portfolios on Ken French’s website at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html>

They can be used to estimate the factor betas for any stock.

**More Factors**

In 1997, Mark Carhart proposed adding a fourth factor to account for the observed success of a momentum strategy. This fourth factor has been adopted by many researchers and analysts.

Factor 4 = WML = Winners minus Losers = Return of a portfolio of stocks which have had high returns over the past year minus the return of a portfolio of stocks that have had a low return over that same time period.

Stocks are ranked by their return over the last year. Those in the bottom 30% are shorted with the proceeds used to purchase those in the top 30%.

In a 2015 paper, Fama and French presented a five-factor model. Based on data from July 1963 – December 2013 (note that this includes an additional 20 years of data that had not yet occurred when they presented their three-factor model in 1992), they still find that their three factors (Rm – Rf, SMB and HML) have explanatory power. But they add two more factors (they do not add WML for momentum):

Factor 4: RMW – the difference between the returns on portfolios of stocks with robust (R) and weak (W) profitability. Profitability is measured with accounting data; Annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses (SGA), all divided by book value of equity (for proper scaling).

R and W stocks are formed the same way H and L stocks are formed (breaks at the 30th and 70th percentiles for NYSE stocks). The idea is that stocks with robust profitability tend to outperform stocks with weak profitability (just as value stocks tend to outperform growth stocks). Again, Fama and French theorize that this factor is proxying for some risk factor in the stocks.

Factor 5: CMA – the difference between the returns on portfolios of stocks of low and high investment firms (where C stands for conservative and A stands for aggressive). Investment is measured as the growth of total assets during the year divided by total assets at the beginning of the year (percentage growth in assets during the year).

Again, C and A stocks are formed with breaks at the 30th and 70th percentiles for NYSE stocks. Interestingly, the conservative stocks – the ones with less investment in total assets tend to outperform the aggressive stocks – the ones with more investment in total assets.

Researchers and analysts who want to include the momentum factor (WML), now make it a six-factor model. Fama and French do not include the momentum factor because they believe that it is purely a happenstance of this particular time period, does not proxy for a risk factor, and is unlikely to continue in the future.