Understanding Risk and Return, the CAPM, and the Fama-French Three-Factor Model

RISK AND RETURN

The General Concept: Higher Expected Returns Require Taking Higher Risk

Most investors are comfortable with the notion that taking higher levels of risk is necessary to expect to earn higher returns. In this note, we explain two important models that have been developed to make this relationship precise. Then we explain how such tools can be used by investors to evaluate assets such as mutual funds.
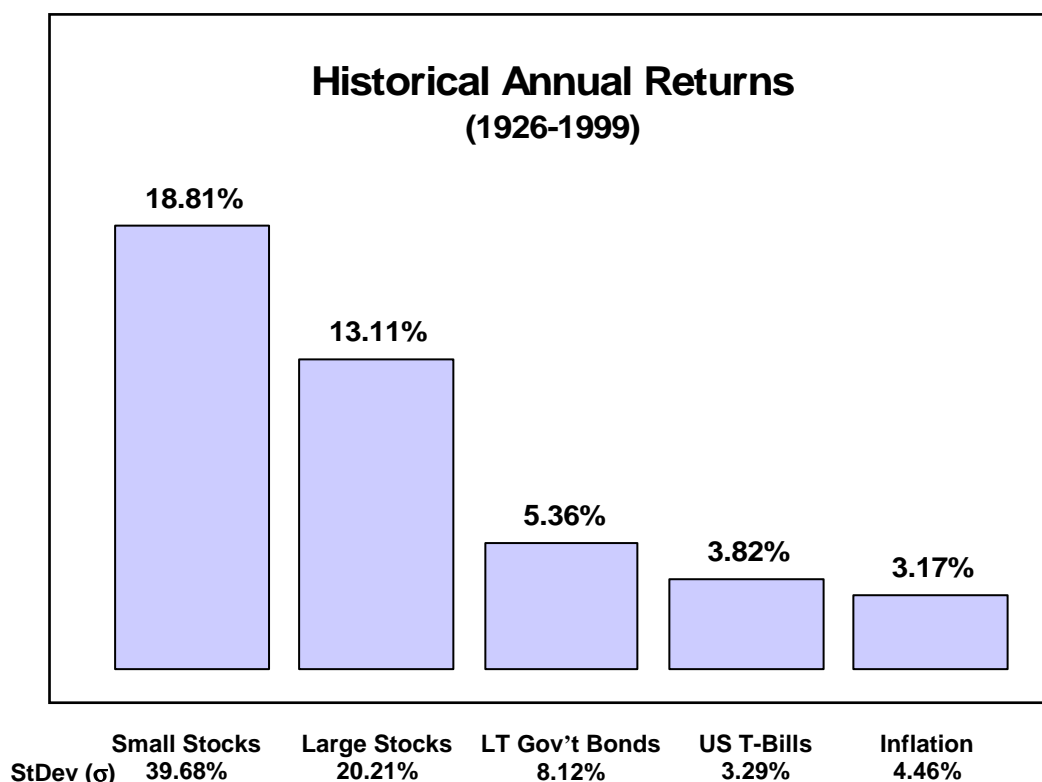
Why should riskier companies have higher returns? Intuitively, an investor would require a higher *expected* return in exchange for accepting greater risk. And, we do, in fact observe this relationship when we look back at historical long-run returns of stocks, bonds, and less risky securities as shown in the first chart.

To understand this, imagine an investment that is expected to generate $1 million per year in perpetuity. How much is someone likely to pay for such an asset? The answer depends on the uncertainty or riskiness of the cash flows. With complete certainty that the cash flows will all be paid when promised, an investor would discount the asset at the risk-free rate. As the degree of uncertainty increases, the return required to justify the risk will be much higher, resulting in a much lower price the investor would be willing to pay, simply because of the higher required discount rate.

Furthermore, economists have made the assumption that investors are risk-averse, meaning that they are willing to sacrifice some return (and accept even less than the expected present value of the future returns) to reduce risk. If this assumption is true, we would expect investors to demand a higher return to justify the additional risk accepted by holders of riskier assets.

Historical Annual Returns
(1926-1999)

	Small Stocks	Large Stocks	LT Gov't Bonds	US T-Bills	Inflation
Return	18.81%	13.11%	5.36%	3.82%	3.17%
StDev (σ)	39.68%	20.21%	8.12%	3.29%	4.46%

Source: Stocks, Bonds, Bills and Inflation 2002 Yearbook, 2002 Ibbotson Associates

Volatility as a Proxy for Risk

One widely accepted measure of risk is volatility, the amount that an asset's return varies through successive time periods, and is most commonly quoted in terms of the standard deviation of returns. An asset whose return fluctuates dramatically is perceived to have greater risk because the asset's value at the time when the investor wishes to sell it is less predictable. In addition, greater volatility means that, from a statistical perspective, the potential future values of more volatile assets span a much wider range.

Diversification and Systematic Risk

Although somewhat counterintuitive, an individual stock's volatility in and of itself, is not the most important consideration when assessing risk. Consider a situation in which an investor could, without incurring additional cost, reduce the volatility associated with her portfolio of assets. This is most commonly accomplished through diversification. Consider holding two stocks that have the same expected returns, instead of one stock. Because stock returns will not be perfectly correlated with each other, it is unlikely that both stocks will experience extreme movements (positive or negative) simultaneously, effectively reducing volatility of the overall portfolio. As long as assets do not move in lock step with one another (are less than perfectly

positively correlated), overall volatility can be reduced, without lowering expected returns, by spreading the same amount of money across the multiple assets.

This concept of diversification is one of the main tenets of modern portfolio theory – volatility is reduced through the addition of more assets to a portfolio. It should be noted, however, that the rate of volatility reduction from adding assets decreases as the number of assets in the portfolio increases. As the chart below demonstrates for one potential scenario (20% volatility on each asset and zero covariance between assets), the general rule of thumb is that a portfolio containing 30 or more assets is considered well-diversified.

Diversifying Reduces Portfolio Volatility
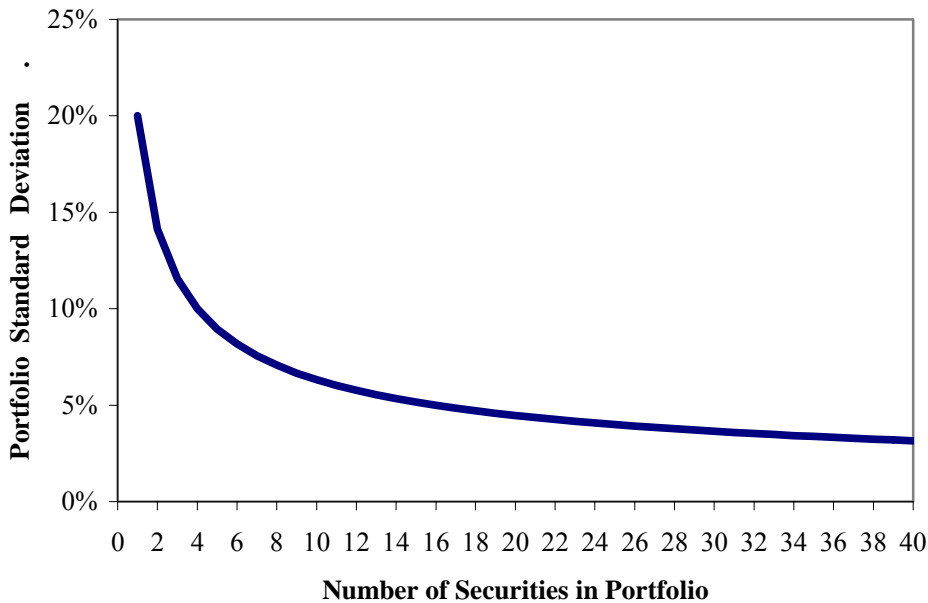
Volatility can be effectively reduced without significant cost by diversifying, so it makes sense that investors should not be compensated for that portion of volatility which is merely stock specific and has no impact on a well diversified portfolio. This type of volatility is called *unsystematic risk* in the finance literature because it does not covary with the market as a whole, but is merely the additional random "noise" present in that specific asset's returns. Since this random noise has an expected return of zero, it can be diversified away by adding more securities to the portfolio. Its mean will be zero, and its standard deviation will be reduced as more assets are added.

The logical extension of this argument is that with enough assets in a portfolio, the portfolio volatility matches that of the overall market. Thus, investors should only expect to be compensated for the risk that cannot be diversified away (i.e. the *systematic risk*).

Beta as a Measure of Systematic Risk
As mentioned above, an asset exhibits both systematic and unsystematic risk. The portion of its volatility which is considered systematic is measured by the degree to which its returns vary relative to those of the overall market. To quantify this relative volatility, a parameter called beta was conceived as a measure of the risk contribution of an individual security to a well diversified portfolio:

$$\beta_A = \frac{\text{cov}(r_A, r_M)}{\sigma_M^2}$$

where
r_A is the return of the asset
r_M is the return of the market
σ_M^2 is the variance of the return of the market, and
$\text{cov}(r_A, r_M)$ is covariance between the return of the market and the return of the asset.

In practice, beta is calculated using historical returns for both the asset and the market, with the market portfolio being represented by a broad index such as the S&P 500 or the Russell 2000. This type of data is widely available from financial databases and can be downloaded into software packages like Excel or SPSS for easy manipulation.

To determine the beta of a portfolio, we simply average the individual securities' betas, weighted by the market capitalization of each security.

The next section describes how such a measure of risk can be used in a model to describe the relationship between systematic risk and expected return.

CAPM

Key Assumptions Drive the Formulation of the Model
The Capital Asset Pricing Model (CAPM) attempts to quantify the relationship between the beta of an asset and its corresponding expected return. The CAPM model makes a number of simplifying assumptions, of which the most relevant to this note are about investor behavior and the presence of a single common risk factor.

The first assumption is that investors care only about expected returns and volatility. Therefore, as rational consumers, they will always maximize expected return for any given level of expected volatility. Second, all investors have homogeneous beliefs about the risk/reward tradeoffs in the market.

The third assumption is that only one risk factor is common to a broad-based market portfolio. This risk factor is the systematic market risk which drives non-diversifiable volatility. Investors are assumed to hold diversified portfolios, as the market does not reward investors for the bearing of diversifiable risk. As a result, the CAPM states that if a security's beta is known, it is possible to calculate the corresponding expected return.

Logic of the Model: Developing Intuition
To build the intuition for this model, first consider an asset that has no volatility, and thus, no risk; thus, its returns do not vary with the market. As a result, the asset has a beta equal to zero and an expected return equal to the risk-free rate.

Next, consider an asset that moves in lock-step with the market, or has a beta of one. As a result of this perfect correlation with the market, this asset, by definition, earns a return equal to that of the market, $E(r_A) = E(r_M)$

Lastly, think about an asset that experiences greater swings in periodic returns than the market, or has a beta greater than one. We would expect this asset to earn returns superior to those of the market as compensation for this extra risk.

If we generalize this relationship between expected returns on assets and their exposure to market risk, we are led to the CAPM equation:

$$E(r_A) = r_f + \beta_A \left(E(r_M) - r_f \right)$$

where r_f is the risk-free rate, and

$(E(r_M) - r_f)$ is the expected excess return of the market portfolio beyond the risk-free rate, often called the equity risk premium.

Essentially, the CAPM states that an asset is expected to earn the risk-free rate plus a reward for bearing risk as measured by that asset's beta. The chart below demonstrates this predicted relationship between beta and expected return – this line is called the Security Market Line.
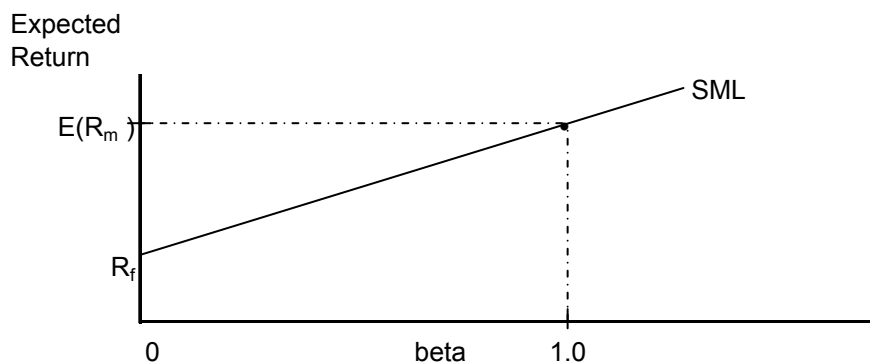
In plain English, beta is the ratio of the expected excess return of an asset relative to the overall market's excess return, where excess return is defined as the return on any given asset less the return on a risk-free asset. For example, a stock with a beta of 1.5 would be expected to have an excess return of 15% in a time period where the overall market beat the risk-free asset by 10%. Effectively, beta is a numerical way to express the idea that expected returns are more sensitive to market swings for those assets that are highly covariant with the market.

The CAPM as a Tool to Evaluate Fund Managers
Given that the CAPM predicts what a particular asset or portfolio's expected return should be relative to its risk and the market return, the CAPM can also be used to evaluate the performance of active fund managers.

Active fund managers try to outperform the market by selecting stocks in a portfolio based on research and informed opinions. One of the key questions surrounding realized returns is whether the manager of the fund is actually achieving a return higher than what would be predicted by the risk the manager took. The CAPM model gives us an estimate of what the return should have been, given the beta risk of the portfolio. If the realized return is greater than the predicted return from the CAPM model, this points toward "adding value;" if the manager has lower or equivalent returns, she might be "just collecting fees" but adding no investment value.

Based on our previous discussion of risk/return tradeoffs, we can see that one way for a manager to increase the expected return on a given fund is to invest in positions that embody greater systematic risk. In effect, by accepting more variance, the manager can increase the beta (and thus the portfolio risk) of the fund and thereby increase her expected returns.

While some investors may choose to accept greater risk to increase expected returns, real value comes from a mutual fund manager who is able to deliver higher returns at the same or reduced level of risk. Essentially, we are asking if the manager is able to create a portfolio which would have higher returns than those predicted by the CAPM. Compare the realized return of a portfolio with its expected return predicted by CAPM. The difference is "excess return", which is often referred to as "α" (or, alpha). Graphically, if α is greater than zero, this portfolio would lie above the Security Market Line. The presence or absence of a positive alpha can be used to evaluate a manager's performance.
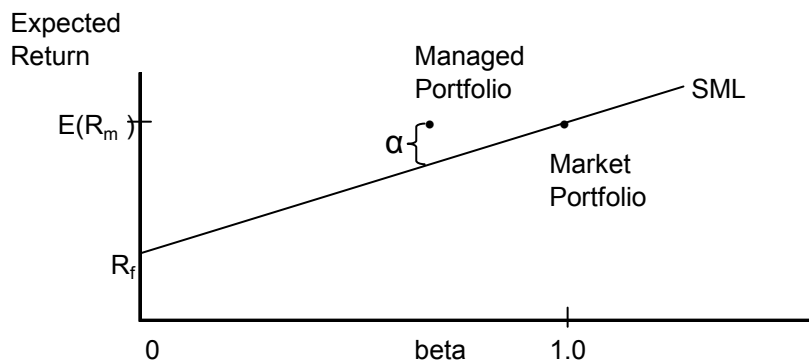
Regression Analysis: A Tool for Employing the CAPM

In order to discern whether a manager should be credited with adding value, we can analyze the manager's portfolio using the CAPM model and regression.

In our case, we would like to know how the return on a particular asset or portfolio changes with respect to the return of the market. We need three time series of data to run this regression. First, we need returns (usually monthly) for the stock whose beta we are calculating for a significant period of time (often 3 or 5 years). Second, we need returns on the overall market index for the same period. Finally, we need risk free returns for the same time period as well. Not surprisingly, the equation looks very similar to the CAPM equation introduced above:

$$r_A = r_f + \beta_A(r_M - r_f) + \alpha$$

Note the addition of alpha, to represent the potential value addition of a fund manager. Furthermore, notice that the beta term in the regression formula is equivalent to the beta term introduced earlier, and is calculated in the same manner.

By rearranging the terms slightly, we will be able to run a regression and determine whether α is indeed reliably positive or not. To run the test, we set up the data as excess returns, subtracting the r_f term from both sides of the equation.

$$r_A - r_f = \alpha + \beta_A(r_M - r_f)$$

Now the equation takes the familiar form of a linear model and we can regress historically realized excess fund (or, individual stock) returns against historically observed excess market returns. Effectively, regression takes a scattered set of points on a graph and determines the line which most closely fits those points. Beta is the slope of this line. Alpha, the vertical intercept, indicates how much better the fund did than the CAPM predicted. Graphically, this is shown as:
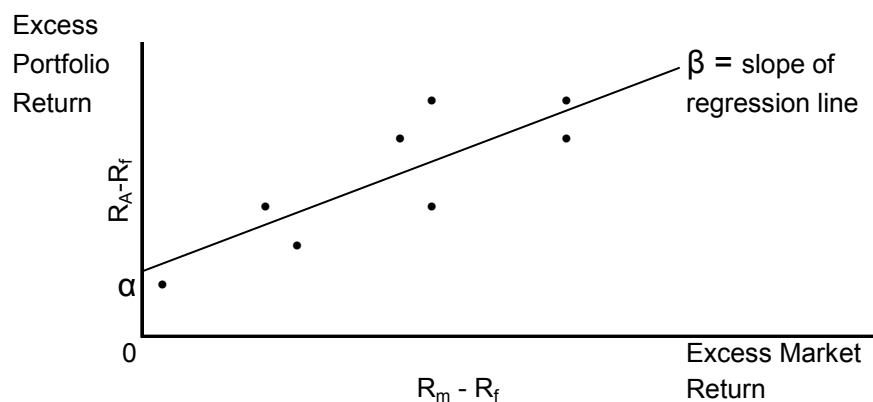
The regression line is expected to pass through the origin if alpha is zero, and alpha can be negative in some cases.

Critique of the CAPM

While the CAPM is an extremely elegant and useful tool, there are concerns about the overall efficacy of the model. Several key criticisms have come to the fore of academic research in recent years:

The CAPM's true predictive power is questionable. When realized returns are compared to what the CAPM would have expected, we find that the model is often incorrect. We find that CAPM models usually achieve an R^2 measure of only about 0.85. While this relatively high R^2 value is one of the main reasons for the popularity of the CAPM, it also highlights the fact that roughly 15% of the variation in observed returns still remains unexplained.

In addition, many researchers believe that other risk factors have significant impact on expected returns in the market. As a result, the simplicity of the CAPM's assumption of a single risk factor explaining expected returns has been called into question.

These critiques are in many ways interrelated; improvements in any one of these areas are bound to have an effect on others. Because the predictive and explanatory power of the CAPM is bound by the structure of the model, it is the assumption of a single risk factor which has spurred much recent academic research into security price analysis.

Additional Factors Increase Predictive Power
It is obvious that there are a myriad of risk factors facing companies today. Some of these factors are market risk, bankruptcy risk, currency risk, supplier risk, etc.; and given that the CAPM uses a single factor to describe aggregate risk, it seems logical that a model including more sub-factors might provide a more descriptive and predictive model. Effectively, additional factors allow more specific attribution of the risks to which a company is exposed. The single risk factor can be decomposed along multiple dimensions.

Furthermore, from a statistical perspective, the addition of independent variables to a regression often improves the explanatory power of a model. For these reasons, multifactor models relax the assumption and constraint of a single risk factor and look for other factors that affect expected return to assets.

As a result of the many hypotheses regarding various risk factors, and the abundance of data available regarding publicly traded stocks, a great deal of research has been performed with the goal of identifying additional risk factors that have robust predictive capability.

FAMA AND FRENCH AND THE THREE FACTOR MODEL

Size and Value Factors Create Additional Explanatory Power
Renowned researchers Eugene Fama and Ken French have done extensive research in this area and found factors describing "value" and "size" to be the most significant factors, outside of market risk, for explaining the realized returns of publicly traded stocks. To represent these risks, they constructed two factors: SMB to address size risk and HML to address value risk. Fama

and French first published their findings on these factors in 1992 and have continued to refine their work since.

The SMB and HML Factors

The SMB Factor: Accounting for the Size Premium

SMB, which stands for Small Minus Big, is designed to measure the additional return investors have historically received by investing in stocks of companies with relatively small market capitalization. This additional return is often referred to as the "size premium."

In practice, the SMB monthly factor is computed as the average return for the smallest 30% of stocks minus the average return of the largest 30% of stocks in that month. A positive SMB in a month indicates that small cap stocks outperformed large cap stocks in that month. A negative SMB in a given month indicates the large caps outperformed. As with the CAPM, when performing historical analysis, we use computed SMB factors for each time period, most commonly monthly; and for predictive purposes (computing an "alpha" excess return), we use either the historical average of the factor or a well informed guess as to the current size premium. As points of reference, the historical average from July 1926 to July 2002 of the annual SMB factor has been approximately 3.3%[1]; and in a recent lecture, Ken French stated that he believes the annual SMB premium to be in the range of 1.5-2.0% today[2].

The HML Factor

HML, which is short for High Minus Low, has been constructed to measure the "value premium" provided to investors for investing in companies with high book-to-market values (essentially, the value placed on the company by accountants as a ratio relative to the value the public markets placed on the company, commonly expressed as B/M).

Constructed in a fashion similar to that of SMB, HML is computed as the average return for the 50% of stocks with the highest B/M ratio minus the average return of the 50% of stocks with the lowest B/M ratio each month. A positive HML in a month indicates that value stocks outperformed growth stocks in that month. A negative HML in a given month indicates the growth stocks outperformed. Over the time period from 1926 to 2002, this premium for value stocks has averaged approximately 5.1% annually[3], and was recently cited by Ken French as having a current value of approximately 3.5-4.0%[4].

Interpretations of the Factors

In reality, the SMB and HML factors first drew attention and continue to be the most commonly used simply because they work—they have the greatest predictive power of any two additional factors that researchers have tested –often yielding an R^2 value of approximately 0.95. That being said, causal explanations for SMB are appealing from a theoretical perspective, but for HML, the labeling of it as a "risk factor" has spurred much discussion.

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[1] Lecture note "The cross-section of expected returns", Investments Course Fall 2003, Ken French
[2] Investments Course Fall 2003 Review Session, November 3, 2003
[3] Lecture note "The cross-section of expected returns", Investments Course Fall 2003, Ken French
[4] Investments Course Fall 2003 Review Session, November 3, 2003

For SMB, which is a measure of "size risk", small companies logically should be expected to be more sensitive to many risk factors as a result of their relatively undiversified nature and their reduced ability to absorb negative financial events.

On the other hand, the HML factor suggests higher risk exposure for typical "value" stocks (high B/M) versus "growth" stocks (low B/M). This makes sense intuitively because companies need to reach a minimum size in order to execute an Initial Public Offering; and if we later observe them in the bucket of high B/M, this is usually an indication that their public market value has plummeted because of hard times or doubt regarding future earnings. Since these companies have experienced some sort of difficulty, it seems plausible that they would be exposed to greater risk of bankruptcy or other financial troubles than their more highly valued counterparts.

Constructing the Three Factor Model
By combining the original market risk factor and the newly developed factors, we have the commonly used Fama French Three Factor Model. Analogous to the CAPM, this model describes the expected return on an asset as a result of its relationship to three risk factors: market risk, size risk, and "value" risk.

$$r_A = r_f + \beta_A (r_M - r_f) + s_A SMB + h_A HML$$

The coefficients in this model have similar interpretations to beta in the CAPM above. β_A is a measure of the exposure an asset has to market risk (although this beta will have a different value from the beta in a CAPM model as a result of the added factors), s_A measures the level of exposure to size risk and h_A measures the level of exposure to value risk.

SMB and HML Provide Added Descriptive Dimensions for Riskiness
A primary implication of the Three Factor Model is that investors can choose to weight their portfolios such that they have greater or lesser exposure to each of the specific risk factors, and therefore can target more precisely different levels of expected return.
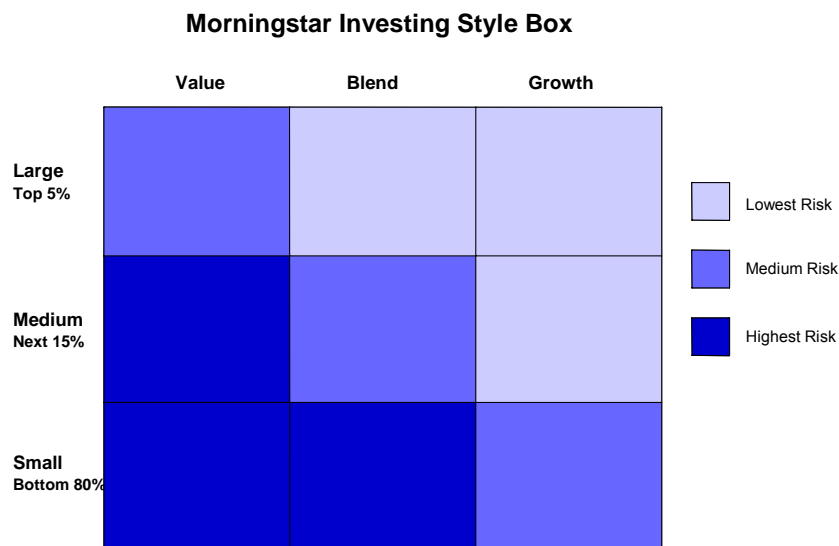
Categorizing Funds with the Three Factor Model
One compelling feature of the Three Factor Model is that it provides a way to categorize mutual funds by the size and value risks to which its portfolio is exposed, and thus the return premiums expected, as a result of the assets held. Utilizing this classification provides two main benefits.

Classifying Funds into Style Buckets
We can effectively compare managers by placing them in broad buckets based on the style of asset allocation they choose in constructing their portfolios. For this purpose, funds are often plotted on a 3x3 matrix, demonstrating the relative amount of risk represented by the different strategies.

The mutual fund rating company Morningstar is the biggest resource for mutual fund classification. Funds are separated horizontally into three roughly equal groups through a B/M ranking (value ranking). Independently, funds are also separated vertically based on a ranking of market capitalization (size ranking), bucketed according to the percentages listed below.

Morningstar Investing Style Box

	Value	Blend	Growth
Large Top 5%			
Medium Next 15%			
Small Bottom 80%			

- Lowest Risk
- Medium Risk
- Highest Risk

Interestingly, the Morningstar classification of a fund is often different from what the fund claims as its official strategy, indicating the value of independent verification.

Specifying Risk Factor Exposure Informs Investor Choice
The second use of the Three Factor Model in categorizing funds is that investors can effectively choose the amount to which they are exposed to each risk factor when investing in particular funds. In practice, this characterization is executed through multivariate regression. The historical returns of a particular portfolio are regressed against the historical values of the three factors, generating estimates of the coefficients.

Funds can then be categorized much more granularly, as presented below:

Three-Factor Model: Manager Profiles
January 1986-December 1995
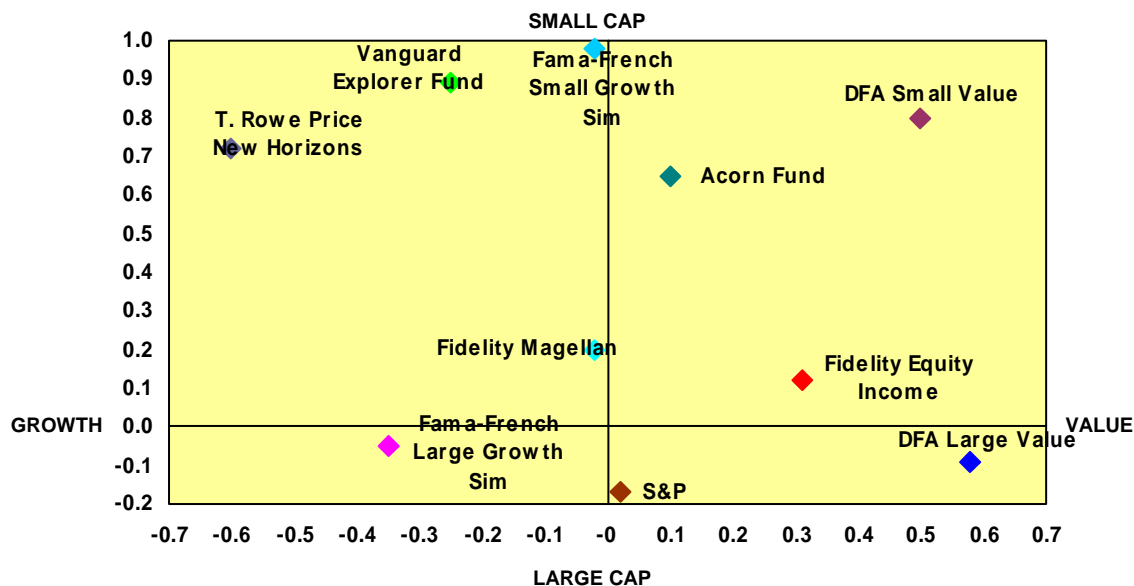
Source: Fama, Eugene. "Asset Management: Engineering Portfolios for Better Returns", *Senior Consultant*, May 1998.

With just these few funds we can see they cover the spectrum of possible strategies described by the Three Factor Model.

Multivariate Regression and Evaluating Managers with the Three Factor Model
Now that we've seen the ability of the Three Factor Model both to classify mutual funds and to demonstrate the ability of investors to choose exposure to certain risk factors, the logical extension is to apply these inferences to the historical performance of fund managers and further refine our ability to determine the amount of value added by management.

In practice, this exercise is merely an extension of the evaluation process described above with respect to the CAPM, but now we need five time series of returns and factors. As mentioned earlier, we first need (usually monthly) returns for the stock whose beta we are calculating for a significant period of time (often 3 or 5 years). Second, we need returns on the overall market index for the same period. Third, we need risk free returns for the same time period as well. Fourth and fifth, we need the calculated factors for SMB and HML for each of the months. We manipulate the Three Factor Model in the same fashion, subtracting the risk-free rate from each side of the equation and introducing the same concept of alpha (i.e., excess return) to yield the equation:

$$r_A - r_f = \alpha + \beta_A \left(r_M - r_f \right) + s_A SMB + h_A HML$$

At this point, we can utilize historical data in a multivariate regression to determine the value of alpha and the statistical likelihood that it is materially different from zero as measured by the relevant t-statistic. A reliably positive measure of alpha would indicate that the mutual fund

manager is adding value to the portfolio, beyond merely allocating investments to provide varying degrees of exposure to the three risk factors.

Ultimately, the benefit of regression with the Three Factor Model is two-fold when compared to the simpler CAPM version. First, the Three Factor Model explains much more of the variation observed in realized returns, displaying R^2 values of 0.95 and higher. Second, the Three Factor Model often exposes the fact that a positive alpha observed in a CAPM regression is merely a result of exposure to either HML or SMB factors, rather than actual manager performance.

Fund Evaluation in Practice (1) – Legg Mason (using CAPM)
The Legg Mason Value Prim fund returned 27.3% annually from September 1982 to December 1986 while the market only returned 21.6%. The fund manager might claim the excess returns were due to her exceptional ability at picking stocks. Armed with the CAPM and regression, we are able to evaluate the fund manager's claim of superior performance.

Using historical monthly values for r_A, r_M and r_f, we can determine the values of α and β using the analysis described above. Using $r_A - r_f$ for the y-values and $r_M - r_f$ for the x-values in a regression, the following coefficients are returned:

$\alpha = 0.46\%$ per month
$\beta = 0.93$

The CAPM considers only one-dimensional market risk, so the realized returns must come from either the fund's exposure to market risk or the value added by the manager. The monthly returns that can be attributed to the manager's ability are captured in alpha. The results imply the fund manager was able to add 46 basis points to the fund's return on a monthly basis or about 5.5% per year above the return expected from a portfolio with a beta of .93. The key question is whether she just got lucky or was really able to add value. The t-statistic associated with alpha is 2.37, indicating that achieving such returns without skill would be extremely unlikely probabilistically. The remainder of the realized returns was due to the fund's exposure to market risk or factors not included in the model. Finally, the R^2 of 0.89 tells us that 89% of the variance of the returns experienced were explained by our model.

It would seem from the results above that the manager had, during that time frame, the ability to increase the fund's return beyond the fund's risk exposure, according to the CAPM.

Note that it is conventional in analyzing securities to use monthly return data, but that there is no specific statistical reason other than simple convenience. If we were to use different time periods in delimiting our analysis we would reach approximately the same results.

Fund Evaluation in Practice (2) – Legg Mason Revisited (using the Three Factor Model)
Using the CAPM, our manager was able to support her claim that she could add 46 basis points monthly. We now have another tool with which to scrutinize this claim. We again utilize historic monthly values to build a regression. In this case, we use r_A, r_M, r_f, SMB, and HML. We can

now regress $r_A - r_f$ against $r_M - r_f$, *SMB* and *HML* to determine the values of α, β, s_A and h_A using the equation described above. The following coefficients result:

$\alpha = 0.22$
$\beta = 0.99$
$s_A = 0.36$
$h_A = 0.22$

The new results imply the fund manager was able to add only 22 basis points on a monthly basis. While this alpha is less than we saw with the CAPM, it would still seem she added significant value on an annualized basis. The relatively low t-statistic of 1.1, however, undermines her claim and indicates that the alpha was more likely to have happened by chance (i.e. it is not statistically different from zero).

The high returns are associated with the fund's exposure to size and value risk rather than the skill of the manager. Finally, the increased R^2 (to 0.92) tells us that the three factors explain all but 8% of the variation in historical returns, further lending credence to the findings.

CONCLUSION

We have examined two tools to help investors understand the risk/reward tradeoff which they face when making investments. We first introduced the CAPM, with its inherent simplicity, linking market covariance risk to expected returns. Its simplicity helps to build intuition around the concept of modeling return as a function of risk. The CAPM's simplicity is also its greatest shortcoming, as the underlying assumptions limit its ability to explain and predict actual returns. The Fama-French Three-Factor Model expands the capabilities of the model by adding two company specific risk factors - SMB and HML. The three factors in concert explain most of the returns due to risk exposure.

Both models have many important uses. Two uses discussed in this note are the ability to categorize investments depending on how their returns vary with different risk factors and to evaluate an active manager's performance independent of her fund's risk exposure. With these tools, investors are able to make more informed investment decisions with respect to personal preference regarding the risk/reward tradeoff.