**Implied Forward Rates**

6-mo T-bill Yield = 5.0%

1-yr T-bill Yield = 5.2%

These are current Bond Equivalent Yield quotes.

The semiannual yields are 2.5% and 2.6% (BEY/2)

If you have funds to invest for one year, which is the better way to go: buy the 1-yr T-bill or buy the 6mo T-bill and reinvest in another 6-month T-bill in 6 months?

The choice should be determined by your expectation of where interest rates will be in six months. To make the best choice, you need to understand how to calculate implied forward rates.

What is the implied forward rate for a 6-mo T-bill 6mo from now?

z1 = 6-mo zero-coupon spot rate

z2 = 1-yr zero-coupon spot rate

Implied forward rate is the rate that gives you the same return at the end of the year no matter if you choose the 1yr T-bill or the 6mo T-bill and roll it over.

Invest $100 in the 1-yr T-bill: FV = PV (1+r)t

 = 100 (1.026)2 = 105.27

Invest $100 in 6-mo T-bill: FV = 100 (1.025) (1 + 1f2) = 105.27

Note: 1f2 is my notation for the forward rate beginning in time 1 and ending in time 2.

The question: What 6-mo rate will take you from $102.5 at 6 months to $105.27 at one year? This rate makes the two paths equivalent, and makes us indifferent between the two bonds.

Substituting and canceling:

(1.026)2 = (1.025) (1 + 1f2)

(1 + 1f2) = (1.026)2 = 1.027 ⇒ 1f2 = .027001 = 2.7001% ⇒ 5.4002% = B.E.Y.

 (1.025)

This is the implied yield on a 6mo T-bill in 6months.

If we are indifferent between the two investments, it’s because we expect the 6-mo rate

6-mo from now to be 5.4%. It is the **Implied Forward Rate**.

The same principle can be used to get any implied forward rate The general formula is:

1 + 1f2 = (1 + z2)2

 (1 + z1)

where z1 and z2 are spot (zero) interest rates.

Suppose we have the following observable interest rates:

|  |  |  |
| --- | --- | --- |
| **Maturity** | **YTM**  | **Semi-annual Yield** |
| 6 months | 2.00% | 1.00% |
| 1 year | 2.50% | 1.25% |
| 18 months | 3.20% | 1.60% |
| 2 years | 4.00% | 2.00% |
| 2.5 years | 4.10% | 2.05% |

We can calculate the implied 6-mo forward rates.

First calculate 1f2: The implied 6-mo forward rate 6 months from now

 (1.0125)2 = (1.01) (1+1f2) ⇒ 1f2 = .015006 ⇒ BEY = 3.0012%

Next let’s look at 2f3 which is the 6-mo forward rate 1 year from now.

(1.016)3 = (1.01) (1.015006) (1+2f3) ⇒ 2f3 = .02304 ⇒ BEY = 4.608%

Now look at 3f4 which is the 6-mo forward rate 1½ yrs from now.

(1.02)4 = (1.01) (1.015006) (1.02304) (1+3f4) ⇒ 3f4= .0321 ⇒ BEY = 6.42%

or you can do it this way:

(1.02)4 = (1.016)3 (1+3f4) ⇒ 3f4 = .0321 ⇒ BEY = 6.42%

We could go on and get all the remaining implied 6-mo forward rates. We could also figure out an implied 1-yr, 2-yr or any other time-period implied forward rates.

Suppose you want to find the implied forward one-year rate one year from now

(1.02)4 = (1.0125)2 (1+2f4)2 ⇒ 2f4 = .0276 ⇒ BEY = 5.52%

The general formula is:

(1+zt)t = (1+z1) (1+1f2) (1+2f3) (1+3f4) … (1+t-1ft)

 = (1+zt-1)t-1 (1+t-1ft)

Note that this is the **implied** forward rate. It is not the same thing as a forward rate. Forward rates are actual tradable derivative contracts. They are the rates that traders are able to lock-in with other traders. These rates are entirely hypothetical – yet they are very important for investment decisions.

The implied forward rate will help you determine whether you should buy a long-term bond or a short-term bond (and roll-over the proceeds). The answer depends on your personal expectation of future rates vs. the implied forward rate.

If implied forward rate is < your prediction, buy short-term and roll over

If implied forward rate is > your prediction, buy long-term and hold.

Implied forward rates are sometimes called hedgeable rates.