**Hedging Interest Rate Risk for a Portfolio**

**Neutral Hedge** – Movements in market interest rates leave the portfolio value (including the hedge instrument) unchanged.

A similar concept is that of **immunization** – which is the process by which a bond portfolio is created to have an assured value at a specific date in the future no matter what interest rates do between the creation of the portfolio and the future date.

Hedging and immunization are much easier to achieve if we assume there will only be parallel shifts in the Yield Curve – Assume that the yield curve never changes shape. Securities of every duration see their yields shift up or down by the same amount at the same time. There is a perfect correlation of changes in the yields of any two securities.

If interest rates go up, the value of your fixed-income asset will go down. However, the future value of your reinvested coupon payments will go up.

If interest rates go down, the future value of your reinvested coupon payemtns will go down, but the value of your fixed-income asset will go up.

If you need to have your asset produce a specified level of money by a certain date, you must consider the trade-off between these two risks: Interest Rate Risk and Reinvestment Risk.

Under the assumption of parallel shifts in the yield curve, matching the modified duration of your assets with the modified duration of your liabilities will provide a neutral hedge and will immunize your portfolio.

Set your portfolio so that the modified duration of your portfolio equals zero.

M.Durp = Σ wi M.Duri = 0

Where wi = Mkt Valuei

Σ Mkt Valuei

Note that long positions (assets) are positive values of wi while short positions (liabilities) are negative values. So, essentially, this just means that you should match the modified durations of your assets with the modified durations of your liabilities. We calculate the duration (and modified duration) of a liability the same way we calculate it for an asset.

Note that we can use Duration just as easily as Modified Duration if we have a flat yield curve (or assume one) because multiplying both sides of the equation by 1+r will switch everything to duration. Now, you can just match the duration of your assets with the duration of your liabilities. This is what is typically done.

The **Duration Gap** of a portfolio tells us if our balance sheet is negatively affected when interest rates go up, or when they go down. We calculate the Duration Gap as:

DURgap = DURA – (L/A ⋅ DURL)

Where L = market value of liabilities and A = market value of assets

Note that in this formula, you would not put a negative sign by your liabilities.

If L = A then,

DURGap > 0 if DURA > DURL

and

DURGap  < 0 if DURL > DURA

L/A ratio allows for presence of more assets or more liabilities.

If Duration Gap is positive, your capital goes down when interest rates go up

If Duration Gap is negative, your capital goes down when interest rates go down

If we wish to relax the assumption of purely parallel shifts in the yield curve, we can no longer use duration to define a portfolio’s price sensitivity to interest rate risk.

If long-term interest rates go up, while short-term interest rates go down (a steepening of the yield curve), two bond portfolios with the same duration may react differently to this change in yields.

Key Rate Duration establishes a bond’s duration measure (responsiveness to changes in the yield) for several different maturities. For example, it may measure how the bond will react to changes in the 2-year rate, the 10-year rate and the 30-year rate. Duration measures for each of them will be set. If the 2-year rate goes down by 50 BP, the 10-year rate is unchanged and the 30-year rate goes up by 70 BP, we will know how our bond will respond.

So if you want to hedge a portfolio by using Key Rate Durtion, you need to match the duration of your assets with the duration of your liabilities at each of these key rates.

Because durations change over time (whether or not there are changes in interest rates), it is necessary to rebalance our portfolio if we wish to remain hedged (immunized).

How often we rebalance depends on the importance of maintaining our hedge and the transactions costs associated with the rebalance.

One other method to immunize a portfolio is to purchase assets specifically for the purpose of matching the cash flows of those assets with the cash flows of your liabilities.

With **cash flow matching**, you start by selecting a bond with the same maturity as your liability with the furthest maturity and where the last cash flow of the bond matches the last cash flow of the liability.

The coupon payments of that first bond are now used to “reduce” the liabilities that match the timing of those coupon payments.

Purchase a second bond which has a maturity equal to the second to the last cash flow of your liabilities and the cash flow of that bond is equal to the “reduced” cash flow of the liability at that time.

Again, use earlier coupon payments to “reduce” the value of the earlier liabilities.

Continue with this process until you have purchased enough bonds so that their combined cash flows match the cash flows of your liabilities.

You probably won’t be able to match the cash flows exactly, and you may well have left-over cash flows from the bonds which don’t match your liabilities. This makes cash-flow matching initially more expensive than immunizing by matching durations, but it will likely require less rebalancing.