**Duration, Modified Duration and Convexity**

Duration is a basic measure of bond price volatility – a measure of interest rate risk.

Modified Duration is the approximate percentage change in price for a 1% change in interest rates

Modified duration does OK for small changes in price (50 BP or less) but (due to a bond’s convexity) the approximation gets poorer as the magnitude of the change increases.

Since a bond’s price sensitivity to changes in the discount rate is influenced by its maturity, yield, and coupon rate, duration should capture all three factors. The traditional measurement of duration; Macaulay Duration does that.

Macaulry Duration is a weighted average of the time periods until a bond matures, weighted by the present value of its cash flows. Though its units are given in years, it is not a measurement of time. It is just helpful to think of it as the fulcrum which balances a timeline that is weighted by the (present value of the) cash flows at each time period.



P = price of the bond

nt = time period t

ct = cash payment at time t

r = discount rate (semiannual yield)

T = periods till maturity

Duration is the sum of the time period multiplied by the present value of the cash flows, divided by the price of the bond.

Example: Calculate the duration of a 2-year bond with an 8% coupon rate with a 10% BEY which is priced at 96.454.



= 3.77 semiannual periods = 1.885 years

Note that the formula gives us duration in semiannual periods, but we must calculate it in terms of years.

Excel has a function for Duration (as well as Modified Duration) where the arguments are the settlement date for the bond, its maturity date, coupon rate, YTM, and frequency (coupon payments per year).

Again, do not be confused by the fact that we are measuring interest rate risk sensitivity in years. Duration is not really a measurement of time, just a convenient way to calculate the value.

For zero coupon bonds, Duration = Maturity of the bond. This makes sense when we think of the bond’s duration as the fulcrum on a see-saw.

Duration is a helpful value, but a more accurate measurement of a bond’s approximate change in price when its yield changes is Modified Duration.

Modified duration is Duration divided by (1 + r).

With annual payments, r = YTM

With semi-ann. payments, r = semi-annual yield (YTM/2)

In our previous example, 1.885/1.05 = 1.7955 = Modified Duration

**Modified Duration** = Approx. percentage change in price of a bond from a 1% change in interest rates.

Second Example:

A bond matures in 6 years, has an 8.4% coupon rate and its YTM is 9.0%

From this information, we can compute the bond’s price as well as its duration either through our formulas, or through Excel.

The bond’s price is $97.2644 and its duration is 4.814

Modified Duration is 4.815/1.045 = 4.607

If the yield changes 1%, price changes approx. 4.607%

If the yield changes 1 BP, price changes approx. 4.607 BP (**.04607%**)

Note this approximation works best for small changes

Let’s calculate the actual changes.

In our example, what if the yield moves from 9% to 9.01%?



= 97.2196 The new price

97.2644 – 97.2196 = .0004606 = **.04606%** = the change in price

97.2644

The price of the bond changes by .04606% in actuality. This is **very close** to the .04607% BP change predicted by modified duration earlier.

Note though that modified duration slightly overestimated the price drop.

Now we’ll try a bigger yield change:

If the yield changes 2%, modified duration predicts a price change of (4.607) (2%) = **9.214%** - Note: You can see this by changing cell H13 in the spreadsheet.

The actual price change is:



= 36.1978 + 52.5982 = 88.7960

97.2644 – 88.7960 = **8.707 %** change - actual price change compared to the 9.214%

97.2644 predicted by modified duration

Note that in both cases, modified duration overstated the change. It predicted a lower price than it actually dropped to. In the first case, it was a very small overstatement, in the second case it was quite significant.

The yield went up so the price went down.

The predicted decline was > the actual decline

A graph will show us why the approx. is less exact for greater yield changes.

Modified Duration is the percentage change in the price of a bond in response to a change in the bond’s yield. Price is the dependent (y) variable and yield is the explanatory (x) variable. We can graph it. It is not a linear relationship.

Note the convex curve. Different bonds have different degrees of convexity. Convexity is good. It limits downside effects and magnifies upside effects.

All else being equal, if you expect interest rates to be volatile, you prefer a bond with greater convexity.

Modified duration is the slope of the tangent line at a particular point on the curve.

The slope will be different at each point on the curve.

The slope of the tangent line is negative.

The slope of the line is the instantaneous change in price for an infinitely small change in the yield.

Modified Duration loses accuracy on larger changes in yield due to the convexity of the curve.

Modified duration is a first-order linear approximation of the change. To be more accurate, we can go to higher order approximations in the Taylor Series.

The second-order term incorporates what we call **Convexity** (or more properly, the

convexity measure).



Since the time periods (n) appear twice in the equation, when we have semiannual compounding, we must divide by four.

If we go back to our first example of the 2-year bond, we can calculate its convexity:



= 16.75817

Dividing by 4 gives us 16.75817 = 4.18954 for our convexity measure

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The convexity measure is always positive

Adding the convexity measure to modified duration improves the fit

Note that for an increase in yield, the price estimate is high.

And for a decrease in yield, the price estimate is low.

There is an inverse relationship between bond price and yield.

It is not linear. It is convex.

So a percentage increase in price when the yield decreases is greater than a percentage decrease in price when the yield increases.

Duration measures the change in price for a given change in yield based on the yield at the time. Modified duration is a first-order approximation of this change. Modified Duration plus convexity is a second-order approximation

To find the approximate price change using both modified duration **and** convexity:

Price Δ ≈ [(-MD) (yld Δ)] + [1/2 (convexity measure) (yld Δ)2]

Going back to our example of the 2-year bond, we calculated that:

Duration = 1.885

Modified Duration = 1.7955

Convexity Measure = 4.18954

Price Δ ≈ [(-1.7955) (.01)] + [1/2 (4.18954) (.01)2] = 1.7745%

So, for a 1% increase in the yield (enter 1.00% in cell H1 of the spreadsheet), modified duration plus convexity predicts a 1.7745% drop in the price (cell N11).

This is much closer to the true price drop of 1.7747% than modified duration alone predicted (Modified Duration predicted 1.7955%).

**Duration of a Portfolio**

The duration of a portfolio is a weighted average of the durations of each security in the portfolio – weighted by each security’s proportion of the total market value of the portfolio.

Note that a bigger portfolio will not necessarily result in a larger duration. The duration of the portfolio is a weighted average of the durations of the individual securities. This makes sense since duration is a measure of the percentage change rather than the dollar change.

Example: You have three assets with the following market values and durations:

Asset 1 - $100 million Duration = 6

Asset 2 - $50 million Duration = 10

Asset 3 - $30 million Duration = 12

Market Value of portfolio = $180 million

Duration of portfolio = (100/180) (6) + (50/180) (10) + (30/180) (12)

= 8.111

**Effective Duration**

These measures of duration, modified duration and convexity only apply to option-free bonds (non-callable, non-puttable, non-convertible, no prepayment of principal, etc.).

For a bond with an embedded option, when interest rates change, there can be a significant change in the expected cash flows for the bond.

For example, for a callable bond, when interest rates go down, it becomes more likely that the bond will be called, and thus the expected cash flows are reduced.

Duration as we have measured it, does not consider any changes in expected cash flows.

Effective Duration does.

Effective duration simply looks at the change in the price of a bond if interest rates go up by some amount and the change in the price of a bond if interest rates go down by the same amount. The absolute value of those price changes are averaged together and expressed as a percentage of the bond’s current price.

Effective Duration = Price if yields decline – Price if yields rise

2 (initial price) (change in yield in decimal)

Note that if there is an embedded option that leads to a change in the cash flows, this will be reflected in the prices in the numerator of the equation. If the bond has no embedded options, the effective duration will equal the duration we calculated earlier.

There is some subjectivity to calculating effective duration .

How much is should we change (shock) the yield? Small changes will give different values from large changes. You will have an exact measure if yields actually change by the amount you used as the shock in the equation, but an approximation for changes that are more or less than that amount.