**Bootstrapping**

The Term Structure of Interest Rates = the relationship between the yield (YTM) and the maturity of bonds with **all other factors held constant**.

Yield Curve = Graph of the term structure

Generally shown with Treasuries since other bonds have too many other variables to be comparable

If the YTM of a 10-year Treasury is 5.52%, this is not necessarily the rate an investor expects to get on a 10-year risk-free investment (with no payments prior to 10 years from now).

The YTM is (sort of, but not exactly) an average of the zero-coupon yields on the cash flows associated with this bond.

The cash flows associated with this bond are the face value (same for all Treasuries) and the coupon payments (not the same for all treasuries).

We need to construct a zero-coupon yield curve that shows the yields investors require on a **single** payment at each specific time period (a series of zero-coupon bonds).

Yields on treasury strips can be used, but because of liquidity differences between Treaury strips and Treasury bonds, bootstrapping is the preferred method.

Example:

Suppose you are a bond trader and you want to buy a 10 yr 10% New Orleans G.O. Municipal Bond. This bond isn’t traded frequently, so you decide to price it based on the price of a comparable Treasury note.

You determine that the tax advantages exactly offset the added riskiness and liquidity constraints of this bond vs. a Treasury note (I do this for convenience).

Thus, you just need to look up the price of a 10 yr 10% Treasury note. But, there are no Treasuries with a maturity of 10 yrs that have a 10% coupon rate. You know that you can’t price it the same as an 8% bond, nor the same as a 9 or 11 yr. bond.

You must derive the price of what a 10% 10 yr Treasury note would be if one existed (a synthetic bond).

The price equals the PV of all the discounted CF.

The cash flows are easy: $5 every 6 months with $105 at the end of 10 yrs.

But what discount rate do we use?

We need to view this coupon bond as a series of zero-coupon bonds.

Each cash flow must be discounted at the appropriate rate for that period

The appropriate rate is the rate for a zero-coupon T-bond for each period.

We must derive the rate for a zero-coupon bond for each period from the prices and yields of bonds that are being traded.

Start with the attached table. They are the prices and yields for actively traded Treasuries as of today. For each maturity, we use the most recently issued Treasury. Notice the coupon rates are all different. We have put together a series of bonds maturing at different times – every 6 mo. – for 10 yrs. The 6 mo and 1 yr T-**bills** pay no coupons – they are discount bonds. All YTMs/BEYs are the semi-annual yields doubled.

Since the 6 mo and 1 yr T-bills are already zero coupon Treasuries, we don’t need to calculate them – just use 8% and 8.3% (note these are BEYs). But the 1.5-yr 8.9% YTM is not a zero-coupon rate. We need to come up with one.

The price of this 1.5-yr coupon bond must equal the PV of the CF, each discounted at the appropriate (zero-coupon) rate.

* 1. = 4.25 + 4.25 + 104.25

 1+z1 (1+z2)2 (1+z3)3

where zi = the appropriate discount rate = the zero-coupon rate for period i

We know z1 and z2 from the 6 mo and 1 yr T-bills so:

* 1. = 4.25 + 4.25 + 104.25

 1.04 (1.0415)2 (1+z3)3

Note:

1 + (8%/2) = 1.04 and 1 + (8.3%/2) = 1.0415

We now have one equation and one unknown

Solving for z3 we get: .04465 = 6mo rate for 1.5-yr zero-coupon bond

Multiply .04465 times 2 to get the Bond Equivalent Yield of .0893 = 8.93%

Note that this is slightly higher than the 8.9% yield of the 8.5% coupon bond.

We can derive all the theoretical spot rates in the same way.

Price of 2-yr bond = 9/2 + 9/2 + 9/2 + 100 + 9/2

 1+z1 (1+z2)2 (1+z3)3 (1+z4)4

 99.64 = 4.5 + 4.5 + 4.5 + 104.5

 1.04 (1.0415)2 (1.04465)3 (1+z4)4

z4 = .04624

Price of 2.5-yr bond = 11/2 + 11/2 + 11/2 + 11/2 + 100 + 11/2

 1+z1 (1+z2)2 (1+z3)3 (1+z4)4 (1+z5)5

 103.49 = 5.5 + 5.5 + 5.5 + 5.5 + 105.5

 1.04 (1.0415)2 (1.04465)3 (1.04624)4 (1+z5)5

z5 = .04734

We continue till we get all the z values (1-20). These z-values make up the zero-coupon Yield Curve.

It gives us the correct discount rates to use to price what a 10yr 10% Treasury-bond would be if it existed

P = 5.0 + 5.0 + 5.0 + 5.0 + … + 105.0

 1.04 (1.0415)2 (1.04465)3 (1.04624)4 (1.06812)20

P = 85.35477

When we total the PV of the discounted CF, we get a price of the 10yr 10% T-Bond of 85.35477.

Now we can price our New Orleans G.O. Muni.

In our example, the N.O. bond should be priced the same as a T-bond (85.35477).

But suppose we feel it should be prices with a 50 b.p. credit spread above the T-bond yield.

First – use the price of the T-bond to calculate its yield. We get 12.62%.

Next – add 50 b.p. to that yield to get 13.12%. Now that we know the yield of the N.O. bond, we can easily find its price which is 82.89275

**Current Data on 20 Different Treasury Securities**

|  |  |  |  |
| --- | --- | --- | --- |
| **Maturity (years)** | **Coupon rate (%)** |  **Yield** **(%)** | **Price** **($)** |
| 0.5 | NA |  8.0 | 96.15 |
| 1.0 | NA |  8.3 | 92.19 |
| 1.5 |  8.5 |  8.9 | 99.45 |
| 2.0 |  9.0 |  9.2 | 99.64 |
| 2.5 | 11.0 |  9.4 |  103.49 |
| 3.0 |  9.5 |  9.7 | 99.49 |
| 3.5 | 10.0 | 10.0 |  100.00 |
| 4.0 | 10.0 | 10.4 | 98.72 |
| 4.5 | 11.5 | 10.6 |  103.16 |
| 5.0 |  8.75 | 10.8 | 92.24 |
| 5.5 | 10.5 | 10.9 | 98.38 |
| 6.0 | 11.0 | 11.2 | 99.14 |
| 6.5 |  8.5 | 11.4 | 86.94 |
| 7.0 |  8.25 | 11.6 | 84.24 |
| 7.5 | 11.0 | 11.8 | 96.09 |
| 8.0 |  6.5 | 11.9 | 72.62 |
| 8.5 |  8.75 | 12.0 | 82.97 |
| 9.0 | 13.0 | 12.0 |  104.30 |
| 9.5 | 11.5 | 12.4 | 95.06 |
|  10.0 | 12.5 | 12.5 |  100.00 |

**Maturity and YTM for 20 Existing Treasury Securities**

**Along with Bootstrapped Zero-Coupon Rates**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Maturity (years)** | **Coupon rate (%)** |  **Yield** **(%)** | **Price** **($)** | **Bootstrapped Semiannual****Zero-Coupon Rate (%)** | **Bootstrapped****BEY** **Zero-Coupon Rate (%)** |
| 0.5 | NA |  8.0 | 96.15 |  4.0 |  8.0 |
| 1.0 | NA |  8.3 | 92.19 |  4.15 |  8.3 |
| 1.5 |  8.5 |  8.9 | 99.45 | 4.465 | 8.93 |
| 2.0 |  9.0 |  9.2 | 99.64 | 4.624 |  9.247 |
| 2.5 | 11.0 |  9.4 |  103.49 | 4.734 |  9.468 |
| 3.0 |  9.5 |  9.7 | 99.49 | 4.894 |  9.787 |
| 3.5 | 10.0 | 10.0 |  100.00 | 5.065 | 10.129 |
| 4.0 | 10.0 | 10.4 | 98.72 | 5.296 | 10.592 |
| 4.5 | 11.5 | 10.6 |  103.16 | 5.425 |  10.85 |
| 5.0 |  8.75 | 10.8 | 92.24 | 5.511 | 11.021 |
| 5.5 | 10.5 | 10.9 | 98.38 | 5.588 | 11.175 |
| 6.0 | 11.0 | 11.2 | 99.14 | 5.792 | 11.584 |
| 6.5 |  8.5 | 11.4 | 86.94 | 5.872 | 11.744 |
| 7.0 |  8.25 | 11.6 | 84.24 | 5.996 | 11.991 |
| 7.5 | 11.0 | 11.8 | 96.09 | 6.203 | 12.405 |
| 8.0 |  6.5 | 11.9 | 72.62 | 6.139 | 12.278 |
| 8.5 |  8.75 | 12.0 | 82.97 | 6.273 | 12.546 |
| 9.0 | 13.0 | 12.0 |  104.30 | 6.576 | 13.152 |
| 9.5 | 11.5 | 12.4 | 95.06 | 6.689 | 13.377 |
|  10.0 | 12.5 | 12.5 |  100.00 | 6.812 | 13.623 |

**Bootstrapping in Excel**

There is no prepackaged formula in Excel for bootstrapping, so we must put one together ourselves. I encourage you to try this on your own. All it takes is some basic algebra and excel skills (and some time and patience).

However, here are a few helpful hints:

The term “discount factor” means 1 or 1 or 1 etc.

 1+z1 (1+z2)2 (1+z3)3

So “sum of the discount factors” = 1 + 1 + 1 etc.

 1+z1 (1+z2)2 (1+z3)3

If we have a bond maturing in 1.5 years (3 semiannual periods from now):

P = C + C + C+100

 1+z1 (1+z2)2 (1+z3)3

Which can be rearranged as:

(1+z3)3 = 

and

z3 = 

If we pull out the Cs in the denominator, we get:

z3 = 

Notice the sum of the discount factors in the denominator. We can use this to simplify our work in excel.